DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 09-13

Search, Heterogeneity, and Optimal Income Taxation

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November 2009

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November 9, 2009

Abstract

[illegible]

D [illegible] . N [illegible] M [illegible] (1982), H [illegible] (1990) [illegible] A [illegible] (1999) [illegible] . H [illegible] (2001) [illegible] . I [illegible]

[illegible] . I [illegible] . I [illegible]

2 Model

(H) (L)

l_k, $k = H; L$, q_m, $m = H; L$.

$y_{km} > 0$,

$y_{Hm} > y_{Lm}$.

$\in 0; 1$,

$c_w(\ _k)$,

$c_w(0) = 0,\ c'_w(0) = 0,$ $\lim_{\delta_k \to 1} c'_w(\ _k) = +\infty$.

v_m, $c_\pi(v_m)$

2.1 The matching technology

, 1 – ff . A M P (1999) , . H , P (1996) B D -

2.2 Output sharing

2.4 Private expected utility functions

fi , , $v_H q_H =$ $_m q_m$.

k

$$U_k = -c_w(\ _k) + \ _k \ M(\ ) \ \frac{v_H q_H}{_m v_m q_m} \ _{kH} y_{kH} + \frac{v_L q_L}{_m v_m q_m} \ _{kL} y_{kL} \ + (1 - M(\ ))0 \ + (1 - \ _k)0$$

$$U_k = -c_w(\ _k) + \ _k M(\ ) E_{(m)} \ _{km} y_{km}; \tag{5}$$

$E_{(m)}$. A , , $c(\ )$, . O , $M(\ )$, . I , $1 - M(\ )$, , , . I , (

3 Optimal search intensity and market ine ciencies

3.1 Social Optimum

$$W = \;_{\delta,v} \quad _k\; l_k U^k + \quad _m\; q_m V^m$$

$$. \quad . \quad _k \geq 0; \quad v_m \geq 0:$$

(1), (2), (5), (6), -91.21

$$E_{(m)}y_{km} - (1 - \quad)E_{(k)}E_{(m)}y_{km} = \quad E_{(k)}E_{(m)}y_{km} + E_{(m)}y_{km} - E_{(k)}E_{(m)}y_{km}:$$

A ... $E_{(k)}E_{(m)}y_{km}$ ... $E_{(m)}y_{km}$ ... $E_{(k)}E_{(m)}y_{km}$.

$$c'_\pi(\bar{v}_H) = \frac{M(\bar{\ })}{\bar{\ }} E_{(k)}y_{kH} - \quad E_{(k)}E_{(m)}y_{km} \qquad \bar{\ } \lesseqgtr 1; \qquad (13)$$

$$c'_\pi(\bar{v}_L) = \frac{M(\bar{\ })}{\bar{\ }} E_{(k)}y_{kL} - \quad E_{(k)}E_{(m)}y_{km} \qquad \bar{v}_H > 0;\ \bar{v}_L > 0$$

$$c'_\pi(\bar{v}_H) = \frac{M(\ )}{\ } E_{(k)}y_{kH} - \quad E_{(k)}E_{(m)}y_{km} \qquad \bar{\ } \lesseqgtr 1; \qquad (14)$$

$$c'_\pi(0) \geq \frac{M(\ )}{\ } E_{(k)}y_{kL} - \quad E_{(k)}E_{(m)}y_{km} \qquad \bar{v}_H > 0;\ \bar{v}_L = 0$$

3.2 Decentralized equilibrium

A ... k ...

$$\max_{\delta_k} \quad U_k = -c_w(\ _k) + \ _k M(\ )E_{(m)}\ _{km}y_{km} \qquad (15)$$

$$\text{s. t.} \quad _k \geq 0;$$

... $C_k = c_w(\ _k)$ ... $B_k = \ _k M(\ )E_{(m)}\ _{km}y_{km}$ ...

K [12]

$$-c'_w(\ _k) + M(\ )E_{(m)}\ _{km} y_{km} \le 0$$

$$_k \ge 0 \qquad (16)$$

$$(-c'_w(\ _k) + M(\ )E_{(m)}\ _{km} y_{km})\ _k = 0;$$

$_k$. , $M(\ )$, . F ,

ff ,

ff .

I -

. $_{LL} \neq 0$ ($_{LH} = 0$), $y_{km} > 0$,

fi (06)2 158.730 ( ) -395. 4.9812163(0),)-373( )-333(( )-350(5. 4

[illegible]

[13] A high type worker, for example, has to search less intensively than a low type worker to achieve the same expected income as a low type worker.

4. *I*

$$c'_w(\ _k) = M(\ )(1 - \ ^w_k)w_k \qquad \lesseqgtr 1;$$
$$c'_\pi(v_m) = \frac{M(\ )}{}(1 - \ ^\pi_m)\ _m \qquad ; \qquad _k > 0;\ v_m > 0 \tag{22}$$

$$c'_w(0) \geq M(\ )(1 - \ ^w_L)w_L \qquad \lesseqgtr 1;$$
$$c'_\pi(0) \geq \frac{M(\ )}{}(1 - \ ^\pi_L)\ _L \qquad : \qquad _L = 0;\ v_L = 0 \tag{23}$$

4.1 Characterizing externalities through Pigou taxes

P ... $(\check{\ }^w_H; \check{\ }^w_L)$, P ... $(\check{\ }^\pi_H; \check{\ }^\pi_L)$. L ... (...) $\check{R}$, P ..., LS:

$$\check{R} = (\ _k\ l)M(\ )\ \frac{_H l_H}{_k\ l}\check{\ }^w_H w_H + \frac{_L l_L}{_k\ l}\check{\ }^w_L w_L + \frac{v_H q_H}{_m vq}\check{\ }^\pi_H\ _H + \frac{v_L q_L}{_m vq}\check{\ }^\pi_L\ _L$$

$$0 = \check{R} - \ _k l_k + \ _m l_m \ LS;$$

$(\ _k\ l)M(\ ) = N$ ... . k

$$U_k = -c_w\ \frac{z^w_k}{M(\ )\,w_k} + LS + (1 - \check{\ }^w_k)z^w_k \tag{24}$$

LS, ... ff ... . fi ... ,

c'_w

[illegible] ([illegible] P [illegible] 5)

[illegible]. CR [illegible], [illegible]; [illegible], [illegible]. [illegible], P [illegible], [illegible]. [illegible], [illegible] ff [illegible]. I[illegible] fi[illegible], [illegible] fi[illegible], L, [illegible].

N [illegible], [illegible] ff [illegible] $(g = 0)$. [illegible], [illegible]. I[illegible]

$y_{LL} > 0$, [illegible]

[illegible]

4.2 Optimal income taxes with positive government revenue

I[illegible] I [illegible] F [illegible] I [illegible] R, [illegible] [17].

[illegible]

$$W = \sum_k l_k U^k + \sum_m q_m V^m ;$$

[illegible]

[illegible] (1), (2), (5), [illegible] (6), [illegible] $w_k = E_{(m)} \; {}_{km} y_{km}$, $z_k^w = {}_k M(\;) w_k$, ${}_m = E_{(k)}(1 - {}_{km}) y_{km}$, [illegible] $z_m^\pi = v_m \frac{M(\theta)}{\theta} {}_m$ [illegible] 4, [illegible]

$$W = \sum_k l_k \left(-c_w \frac{z_k^w}{M(\;) w_k} \right) + \sum_m q_m \left(-c_\pi \frac{z_m^\pi}{\frac{M(\theta)}{\theta} {}_m} \right) + (\textstyle\sum_k l) M(\;) E_{(k)} E_{(m)} y_{km};$$

[illegible] $(\sum_k l) M(\;) = N$ [illegible].

I [illegible] R, [illegible]

$$R \leq (\textstyle\sum_k l) M(\;) \left(\frac{{}_H l_H}{\sum_k l} {}_H^w w_H + \frac{{}_L l_L}{\sum_k l} {}_L^w w_L + \frac{v_H q_H}{\sum_m vq} {}_H^\pi {}_H + \frac{v_L q_L}{\sum_m vq} {}_L^\pi {}_L \right); \tag{30}$$

[illegible] $(\sum_k l) M(\;) = M$ [illegible] (30), [illegible]

[17] The public good, even if valued by consumers, does not a ect their choice on search intensity.

$$W = \sum_k l_k \left(-c_w \frac{z_k^w}{M(\;)\, w_k} \right) + \sum_m q_m \left(-c_\pi \frac{z_m^\pi}{\frac{M(\theta)}{\theta}\,_m} \right)$$

$$+ (\textstyle\sum_k l) M(\;) \left(\frac{_H l_H}{\sum_k l}(1 - {}^w_H) w_H + \frac{_L l_L}{\sum_k l}(1 - {}^w_L) w_L \right.$$

$$\left. + \frac{v_H q_H}{\sum_m vq}(1 - {}^\pi_H)\,_H + \frac{v_L q_L}{\sum_m vq}(1 - {}^\pi_L)\,_L \right) + R.$$

fi ... (22) (C (1978), D (1998)).

H

L 7 A . E , fi (22)

$$\max_{\tau_k^w, \tau_m^\pi} W = \sum_k l_k \left(-c_w \frac{z_k^w}{M(\;)\, w_k} \right) + \sum_m q_m \left(-c_\pi \frac{z_m^\pi}{M(\theta)} \right.$$

[illegible] P [illegible] 8 [illegible] A [illegible]. [illegible]

[illegible] $''^{w}_{k} = 0$ [illegible] $''^{\pi}_{m} = 0$. [illegible] (33). [illegible] H [illegible] P [illegible] 5 [illegible] A [illegible] A). H [illegible]

I [illegible] $''^{\pi}_{m} = \infty$, [illegible] $''^{\pi}_{m} = \infty$, [illegible]

4.2.2 [illegible]

[illegible] I [illegible] 1 [illegible]

/), D (1998)). fi ’ (P 11), , . P 11 ( )

. A

P 5,

12.

$$i) \; \partial \frac{w_H}{w_L} = \partial \frac{E_{(m)} \; Hm}{E_{(m)} \; Lm} < 0 \tag{36}$$

$$ii) \; \partial \frac{w}{\pi} = \partial (\;) < 0 \tag{37}$$

P 11 A . B

(P 5).

P 12

B B (2002)

5 Conclusion

[illegible]

[illegible]:

[illegible] (1) "H [illegible] Effi[illegible] F[illegible]," *I [illegible] E[illegible]*, 40, 827-849.

[illegible] (2002) "O[illegible] L[illegible]," *J[illegible] E[illegible]*, 85, 53-97.

[illegible] (1 4) "Eff[illegible] B[illegible]fi[illegible] F[illegible]," [illegible] P[illegible].

[illegible] (1) "M[illegible] Cl[illegible]," [illegible] *J[illegible] E[illegible]*, 112, 141-168.

[illegible] (1) "L[illegible] - [illegible] P[illegible] F[illegible]: M[illegible] E[illegible]," *E[illegible] J[illegible]*, 109, 307-334.

[illegible] (1) "O[illegible] R[illegible]: M[illegible] R[illegible]," *A[illegible] E[illegible]*, 68, 756-768.

[illegible] (1 1) "M[illegible] C[illegible], F[illegible] Effi[illegible]," *J[illegible] E[illegible]*, .89, 798-812.

[illegible] (1 2) "A[illegible] D[illegible] M[illegible] E[illegible]," *J[illegible] E[illegible]*, .90, 881-894.

[illegible] (1 2) "[illegible] D[illegible] Effi[illegible] E[illegible]," *E[illegible]*, .49, 217-227.

[illegible] (1) "O[illegible] I[illegible]: A[illegible] E[illegible] P[illegible] O[illegible] M[illegible] R[illegible]," *A[illegible] E[illegible]*, .88, 83-95.

[illegible] (1) "[illegible] P[illegible]," *E[illegible] J[illegible]*, 7, 46-70.

[illegible] (1 3) "O[illegible] O[illegible] P[illegible] I[illegible]," *J[illegible] E[illegible]*, 2, 357-376.

[illegible] (1) "H[illegible] D[illegible] P[illegible] Aff[illegible] E[illegible]? C[illegible] O[illegible] E[illegible]," [illegible] M[illegible] P[illegible].

. (1 0) "Effi [illegible] I [illegible]," *J [illegible] E [illegible]*, 51, 8999.

(1 5) "P [illegible], [illegible] P [illegible], D [illegible] E [illegible].

(1 0) "O [illegible] Effi [illegible] M [illegible] R [illegible] M [illegible]," *E [illegible]*, 57, 279-298.

(1) " [illegible] F [illegible]," *E [illegible]*, 32, D [illegible] E [illegible].

(1) " P [illegible] G [illegible] E [illegible] P [illegible] M [illegible] B [illegible]," *L [illegible] E [illegible]*, 3, 6580.

(1) "L [illegible] R [illegible], G [illegible] J [illegible] B [illegible] J [illegible]," *E [illegible], C [illegible]*, [illegible] P [illegible].

(1 3) " [illegible]

(1) "L M D Eff , " J E , 33, 457495.

(2000) "A M , " E , 68, 343-369.

(2001) "M , , H " A , 1(2001), I . 1, A . 5.

(1) "P F E P ?," E , 25, 223264.

(1) "O P F L M ," L E , 6, 435452.

(1) "P Effi O N N E ," I : H E , .2.

(1) " - P Effi ," J E , 6, 239262.

(1) " , P P D .," I F , 6, 239262.

(1) " P F E ," I : , .B. (E .), F C ., M M , L .

Appendices:

A Proofs of the main results

Proof of Corollary 3.

F , . - , $_1 \neq$. I $_1 >$, $_H(_1) > _H()$, $_L(_1) > _L()$, $v_H(_1) < v_H()$, $v_L(_1) < v_L()$. H

$)_1$)

CR

F 1 − =

$$\check{R} = N \begin{bmatrix} \frac{\delta_H l_H}{k\,\delta l}(1 - \quad)E_{(k)}E_{(m)}y_{km} - E_{(m)}(1 - \quad_{Hm})y_{Hm} \\ + \frac{\delta_L l_L}{k\,\delta l}(1 - \quad)E_{(k)}E_{(m)}y_{km} - E_{(m)}(1 - \quad_{Lm})y_{Lm} \\ + \frac{v_H q_H}{m\,vq}(1 - (1 - \quad))E_{(k)}E_{(m)}y_{km} - E_{(k)}\quad_{kH}y_{kH} \\ + \frac{v_L q_L}{m\,vq}(1 - (1 - \quad))E_{(k)}E_{(m)}y_{km} - E_{(k)}\quad_{kL}y_{kL} \end{bmatrix}$$

$$= N \begin{bmatrix} \frac{\delta_H l_H}{k\,\delta l}(1 - \quad)E_{(k)}E_{(m)}y_{km} - E_{(m)}(1 - \quad_{Hm})y_{Hm} \\ + \frac{\delta_L l_L}{k\,\delta l}(1 - \quad)E_{(k)}E_{(m)}y_{km} - E_{(m)}(1 - \quad_{Lm})y_{Lm} \\ + \frac{v_H q_H}{m\,vq}(1 - \quad)E_{(k)}E_{(m)}y_{km} - E_{(k)}\quad_{kH}y_{kH} \\ + \frac{v_L q_L}{m\,vq}(1 - \quad)E_{(k)}E_{(m)}y_{km} - E_{(k)}\quad_{kL}y_{kL} \end{bmatrix}$$

$$\check{R} = N\ 1 - (\quad + \quad)$$

[illegible] . I [illegible] ([illegible] L [illegible]).□

Proof of Lemma 7.

I [illegible] , [illegible] k

$$U_k = -c_w(\quad_k) + \quad_k M(\quad)(1 - \quad_k^w)w_k$$

$$= -c_w\ \frac{z_k^w}{M(\quad)w_k} + (1 - \quad_k^w)z_k^w$$

[illegible] w_k

$$\frac{@U_k}{@w_k} = -c'_w \frac{1}{M(\quad)w_k}\frac{@z_k^w}{@w_k} - c'_w \frac{z_k^w}{M(\quad)} \quad -\frac{1}{w_k^2} \quad + \frac{@z_k^w}{@w_k}(1 - \quad_k^w) = c'_w \frac{z_k^w}{M(\quad)w_k} > 0;$$

[illegible] E [illegible] . □

Proof of Proposition 8.

I : $w_k = E_{(m)} w_{km} = E_{(m)} \; {}_{km} y_{km}$

$k = H; L$; $z_k^w = \; {}_k M(\;) w_k$

$k = H; L$; ${}_m = E_{(k)} \; {}_{km} = E_{(k)} (1 - \; {}_{km}) y_{km}$ fi

e $m = H; L$; $z_m^\pi = v_m \; M$

Z

π π

k

l . F fi , l , l

, l ,

$$\frac{dz^w_H}{z^w_H} \quad \frac{1}{"^w_H} + (1- \quad)\frac{_H l_H}{k \quad l} \quad = (1- \quad) \quad E_{(m)} \quad \frac{dz^\pi_m}{z^\pi_m} \quad - \frac{_L l_L}{k \quad l} \quad \frac{dz^w_L}{z^w_L} \quad - \frac{d \quad ^w_H}{1- \quad ^\pi} \qquad ($$

L

$$\frac{dz_H^\pi}{z_H^\pi}\frac{1}{\varepsilon_H^\pi} = \frac{E_{(k)}\left(\frac{dz_k^w}{z_k^w} + \frac{d\tau_L^\pi}{1-\tau_L^\pi} \frac{v_L q_L}{m\, vq}\varepsilon_L^\pi - \frac{d\tau_H^\pi}{1-\tau_H^\pi}\left(1 + \frac{v_L q_L}{m\, vq}\varepsilon_L^\pi\right.\right.}{\Delta_2} \tag{45}$$

$$\frac{dz_L^\pi}{z_L^\pi}\frac{1}{\varepsilon_L^\pi} = \frac{E_{(k)}\left(\frac{dz_k^w}{z_k^w} - \frac{d\tau_L^\pi}{1-\tau_L^\pi}\left(1 + \frac{v_H q_H}{m\, vq}\varepsilon_H^\pi\right. + \frac{d\tau_H^\pi}{1-\tau_H^\pi} \frac{v_H q_H}{m\, vq}\varepsilon_H^\pi\right.}{\Delta_2}; \tag{46}$$

$\Delta_2 = 1 + E_{(m)}\varepsilon_m^\pi$. A (45) (46)

$$E_{(m)} \frac{dz_m^\pi}{z_m^\pi} = \frac{E_{(k)}\left(\frac{dz_k^w}{z_k^w} E_{(m)}\varepsilon_m^\pi - E_{(m)}\left(\varepsilon_m^\pi \frac{d\tau_m^\pi}{1-\tau_m^\pi}\right.\right.}{\Delta_2}; \tag{47}$$

(43) (44)

$$E_{(k)} \frac{dz_k^w}{z_k^w} = \frac{(1- \;)E_{(m)}\left(\frac{dz_m^\pi}{z_m^\pi} E_{(k)}\varepsilon_k^w - E_{(k)}\left(\varepsilon_k^w \frac{d\tau_k^w}{1-\tau_k^w}\right.\right.}{\Delta_1}: \tag{48}$$

F (47) (48) $E_{(m)}\left(\frac{dz_m^\pi}{z_m^\pi}\right.$ $E_{(k)}\left(\frac{dz_k^w}{z_k^w}\right.$

$$E_{(m)} \frac{dz_m^\pi}{z_m^\pi} = -\frac{(\Delta_2 - 1)E_{(k)}\left(\varepsilon_k^w \frac{d\tau_k^w}{1-\tau_k^w}\right. + \Delta_1 E_{(m)}\left(\varepsilon_m^\pi \frac{d\tau_m^\pi}{1-\tau_m^\pi}\right.}{\Delta_1 + \Delta_2 - 1} \tag{49}$$

$$E_{(k)} \frac{dz_k^w}{z_k^w} = -\frac{\Delta_2 E_{(k)}\left(\varepsilon_k^w \frac{d\tau_k^w}{1-\tau_k^w}\right. + (\Delta_1 - 1)E_{(m)}\left(\varepsilon_m^\pi \frac{d\tau_m^\pi}{1-\tau_m^\pi}\right.}{\Delta_1 + \Delta_2 - 1} \tag{50}$$

(49) (43) (44), (50) (45) (46) fi /

:

$$\frac{dz_H^w}{z_H^w}\frac{1}{\varepsilon_H^w} = -\frac{(1- \;)\;(\Delta_2 - 1)E_{(k)}\left(\varepsilon_k^w \frac{d\tau_k^w}{1-\tau_k^w}\right. + \Delta_1 E_{(m)}\left(\varepsilon_m^\pi \frac{d\tau_m^\pi}{1-\tau_m^\pi}\right.}{\Delta_1(\Delta_1 + \Delta_2 - 1)} + \frac{(\Delta_1 + \Delta_2 - 1)\;\; \frac{d\tau_L^w}{1-\tau_L^w}(1- \;)\frac{\delta_L l_L}{k\,\delta l}\varepsilon_L^w - \frac{d\tau_H^w}{1-\tau_H^w}\left(1 + (1- \;)\frac{\delta_L l_L}{k\,\delta l}\varepsilon_L^w\right.}{\Delta_1(\Delta_1 + \Delta_2 - 1)}; \tag{51}$$

$$\frac{dz_L^w}{z_L^w}\frac{1}{\varepsilon_L^w} = -\frac{(1- \;)\;(\Delta_2 - 1)E_{(k)}\left(\varepsilon_k^w \frac{d\tau_k^w}{1-\tau_k^w}\right. + \Delta_1 E_{(m)}\left(\varepsilon_m^\pi \frac{d\tau_m^\pi}{1-\tau_m^\pi}\right.}{\Delta_1(\Delta_1 + \Delta_2 - 1)} + \frac{(\Delta_1 + \Delta_2 - 1)\;\; -\frac{d\tau_L^w}{1-\tau_L^w}\left(1 + (1- \;)\frac{\delta_H l_H}{k\,\delta l}\varepsilon_H^w\right. + \frac{d\tau_H^w}{1-\tau_H^w}(1- \;)\frac{\delta_H l_H}{k\,\delta l}\varepsilon_H^w}{\Delta_1(\Delta_1 + \Delta_2 - 1)}; \tag{52}$$

$$\frac{dz_H^\pi}{z_H^\pi}\frac{1}{{}''^\pi_H} = -\frac{\Delta_2 E_{(k)}\left({}''^w_k \frac{d\tau_k^w}{1-\tau_k^w} \quad + (\Delta_1 - 1)E_{(m)}\left({}''^\pi_m \frac{d\tau_m^\pi}{1-\tau_m^\pi}\right.\right.}{\Delta_2(\Delta_1+\Delta_2-1)}$$

$$+\frac{(\Delta_1+\Delta_2-1) \quad \frac{d\tau_L^\pi}{1-\tau_L^\pi} \quad \frac{v_L q_L}{{}_k\, vq}{}''^\pi_L - \frac{d\tau_H^\pi}{1-\tau_H^\pi}\left(1 + \quad \frac{v_L q_L}{{}_k\, vq}{}''^\pi_L\right.}{\Delta_2(\Delta_1+\Delta_2-1)}; \tag{53}$$

$$\frac{dz_L^\pi}{z_L^\pi}\frac{1}{{}''^\pi_L} = -\frac{\Delta_2 E_{(k)}\left({}''^w_k \frac{d\tau_k^w}{1-\tau_k^w} \quad + (\Delta_1 - 1)E_{(m)}\left({}''^\pi_m \frac{d\tau_m^\pi}{1-\tau_m^\pi}\right.\right.}{\Delta_2(\Delta_1+\Delta_2-1)}$$

$$+\frac{(\Delta_1+\Delta_2-1) \quad -\frac{d\tau_L^\pi}{1-\tau_L^\pi}\left(1 + \quad \frac{v_H q_H}{{}_k\, vq}{}''^\pi_H \quad + \frac{d\tau_H^\pi}{1-\tau_H^\pi} \quad \frac{v_H q_H}{{}_k\, vq}{}''^\pi_H\right.}{\Delta_2(\Delta_1+\Delta_2-1)}; \tag{54}$$

F [illegible] (51)-(54) [illegible] fi [illegible].

$$\begin{aligned}
\frac{dz_H^w}{d\ {}^w_H}\frac{1}{z_H^w} &= \frac{{}''^w_H}{1- \ {}^w_H} \quad \frac{(1- \quad)\frac{\delta_H l_H}{{}_k\,\delta l}{}''^w_H - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}^w_H}\frac{1}{z_L^w} &= \frac{\frac{\varepsilon_H^w}{1-\tau_H^w}\ \frac{\delta_H l_H}{{}_k\,\delta l}{}''^w_L(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ {}^w_H}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_H^w}{1-\tau_H^w}\ \frac{\delta_H l_H}{{}_k\,\delta l}{}''^\pi_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ {}^w_H}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_H^w}{1-\tau_H^w}\ \frac{\delta_H l_H}{{}_k\,\delta l}{}''^\pi_L}{\Delta_1+\Delta_2-1}
\end{aligned} \tag{55}$$

$$\begin{aligned}
\frac{dz_H^w}{d\ {}^w_L}\frac{1}{z_H^w} &= \frac{\frac{\varepsilon_L^w}{1-\tau_L^w}\ \frac{\delta_L l_L}{{}_k\,\delta l}{}''^w_H(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}^w_L}\frac{1}{z_L^w} &= \frac{{}''^w_L}{1- \ {}^w_L} \quad \frac{(1- \quad)\frac{\delta_L l_L}{{}_k\,\delta l}{}''^w_L - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ {}^w_L}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_L^w}{1-\tau_L^w}\ \frac{\delta_L l_L}{{}_k\,\delta l}{}''^\pi_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ {}^w_L}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_L^w}{1-\tau_L^w}\ \frac{\delta_L l_L}{{}_k\,\delta l}{}''^\pi_L}{\Delta_1+\Delta_2-1}
\end{aligned} \tag{56}$$

$$
\begin{aligned}
\frac{dz^w_H}{d\ ^{\pi}_{H}}\frac{1}{z^w_H} &= -\frac{\frac{\varepsilon^{\pi}_{H}}{1-\tau^{\pi}_{H}}\frac{v_H q_H}{_m vq}\,''^{w}_{H}(1-\quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz^w_L}{d\ ^{\pi}_{H}}\frac{1}{z^w_L} &= -\frac{\frac{\varepsilon^{\pi}_{H}}{1-\tau^{\pi}_{H}}\frac{v_H q_H}{_m vq}\,''^{w}_{L}(1-\quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz^\pi_H}{d\ ^{\pi}_{H}}\frac{1}{z^\pi_H} &= \frac{''^{\pi}_{H}}{1-\ ^{\pi}_{H}}\quad\frac{''^{\pi}_{H}\frac{v_H q_H}{_m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz^\pi_L}{d\ ^{\pi}_{H}}\frac{1}{z^\pi_L} &= \frac{\frac{\varepsilon^{\pi}_{H}}{1-\tau^{\pi}_{H}}\frac{v_H q_H}{_m vq}\,''^{\pi}_{L}}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{57}
$$

$$
\begin{aligned}
\frac{dz^w_H}{d\ ^{\pi}_{L}}\frac{1}{z^w_H} &= -\frac{\frac{\varepsilon^{\pi}_{L}}{1-\tau^{\pi}_{L}}\frac{v_L q_L}{_m vq}\,''^{w}_{H}(1-\quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz^w_L}{d\ ^{\pi}_{L}}\frac{1}{z^w_L} &= -\frac{\frac{\varepsilon^{\pi}_{L}}{1-\tau^{\pi}_{L}}\frac{v_L q_L}{_m vq}\,''^{w}_{L}(1-\quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz^\pi_H}{d\ ^{\pi}_{L}}\frac{1}{z^\pi_H} &= \frac{\frac{\varepsilon^{\pi}_{L}}{1-\tau^{\pi}_{L}}\frac{v_L q_L}{_m vq}\,''^{\pi}_{H}}{\Delta_1+\Delta_2-1}\\
\frac{dz^\pi_L}{d\ ^{\pi}_{L}}\frac{1}{z^\pi_L} &= \frac{''^{\pi}_{L}}{1-\ ^{\pi}_{L}}\quad\frac{''^{\pi}_{L}\frac{v_L q_L}{_m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{58}
$$

[illegible]

I [illegible] L [illegible]

$$
\begin{aligned}
\max_{\tau^w_k,\tau^\pi_m} W = &\sum_k l_k \quad -c_w\left(\frac{z^w_k}{M(\ )\,w_k}\right) + \sum_m q_m \quad -c_\pi\left(\frac{z^\pi_m}{\frac{M(\theta)}{\theta}\ _m}\right)\\
&+\ _H l_H c'_w\left(\frac{z^w_H}{M(\ )\,w_H}\right) + \ _L l_L c'_w\left(\frac{z^w_L}{M(\ )\,w_L}\right) + v_H q_H c'_\pi\left(\frac{z^\pi_H}{\frac{M(\theta)}{\theta}\ _H}\right) + v_L q_L c'_\pi\left(\frac{z^\pi_L}{\frac{M(\theta)}{\theta}\ _L}\right) + R\\
&+ (\ _k\ l)M(\ )\left(\frac{_H l_H}{_k\ l}\ ^w_H w_H + \frac{_L l_L}{_k\ l}\ ^w_L w_L + \frac{v_H q_H}{_m vq}\ ^\pi_H\ _H + \frac{v_L q_L}{_m vq}\ ^\pi_L\ _L\right);
\end{aligned}
$$

[illegible] D [illegible]

$$
a = \frac{_H l_H}{_k\ l}\ ^w_H w_H + \frac{_L l_L}{_k\ l}\ ^w_L w_L \quad \text{and} \quad b = \frac{v_H q_H}{_m vq}\ ^\pi_H\ _H + \frac{v_L q_L}{_m vq}\ ^\pi_L\ _L:
$$

$$_H^w$$

$$\frac{@L}{@\ {}_H^w} =$$

$$= \quad {}_k\ l_k \ -\frac{c'_w}{M(\ )\,w_k}\frac{dz_k^w}{d\ {}_H^w} \quad + \quad {}_m\ q_m \ -\frac{c'_\pi}{\frac{M(\theta)}{\theta}\ \ {}_m}\frac{dz_m^\pi}{d\ {}_H^w}$$

$$+\frac{dz_H^w}{d\ {}_H^w}\frac{1}{M(\ )\,w_H}l_H c'_w \quad \frac{z_H^w}{M(\ )\,w_H} \quad + \quad {}_H l_H c''_w \quad \frac{z_H^w}{M(\ )\,w_H} \quad \frac{1}{M(\ )\,w_H}\frac{dz_H^w}{d\ {}_H^w}$$

$$+\frac{dz_L^w}{d\ {}_H^w}\frac{1}{M(\ )\,w_L}l_L c'_w \quad \frac{z_L^w}{M(\ )\,w_L} \quad + \quad {}_L l_L c''_w \quad \frac{z_L^w}{M(\ )\,w_L} \quad \frac{1}{M(\ )\,w_L}\frac{dz_L^w}{d\ {}_H^w}$$

$$+\frac{dz_H^\pi}{d\ {}_H^\pi}\frac{1}{\frac{M(\theta)}{\theta}\ \ {}_H}q_H c'_\pi \quad \frac{z_H^\pi}{\frac{M(\theta)}{\theta}\ \ {}_H} \quad + \nu_H q_H c''_\pi \quad \frac{z_H^\pi}{\frac{M(\theta)}{\theta}\ \ {}_H} \quad \frac{1}{\frac{M(\theta)}{\theta}\ \ {}_H}\frac{dz_H^\pi}{d\ {}_H^\pi}$$

$$+\frac{dz_L^\pi}{d\ {}_L^\pi}\frac{1}{\frac{M(\theta)}{\theta}\ \ {}_L}q_L c'_\pi \quad \frac{z_L^\pi}{\frac{M(\theta)}{\theta}\ \ {}_L} \quad + \nu_L q_L c''_\pi \quad \frac{z_L^\pi}{\frac{M(\theta)}{\theta}\ \ {}_L} \quad \frac{1}{\frac{M(\theta)}{\theta}\ \ {}_L}\frac{dz_L^\pi}{d\ {}_L^\pi}$$

$$+\left[\ {}_k \frac{l_k}{M(\ )\,w_k}\frac{dz_k^w}{d\ {}_H^w} \quad M(\ )+(\ {}_k\ l)M(\ ) \quad \frac{{}_m\ \frac{q_m}{\frac{M(\theta)}{\theta}\pi_m}\frac{dz_m^\pi}{d\tau_H^w}}{{}_k\ l} - \frac{(\ {}_m\ \nu q)\ \ {}_k\left(\frac{l_k}{M(\theta)\,w_k}\frac{dz_k^w}{d\tau_H^w}\right.}{(\ {}_k\ l)^2}\right] \ (a+b)$$

[illegible]

$$+\ (\ {}_k\ l)M(\ )$$

$$ {}_H l_H\, c''_w \;\frac{z_H^w}{M(\ )\, w_H} \quad \frac{1}{M(\ )\, w_H}\frac{dz_H^w}{d\ {}_H^w} + {}_L l_L\, c''_w \;\frac{z_L^w}{M(\ )\, w} $$ [redacted]

$$ +v_H q_H\, c''_\pi \;\frac{z_H^\pi}{\frac{M(\theta)}{\theta}\ {}_H} \quad \frac{1}{\frac{M(\theta)}{\theta}\ {}_H}\frac{dz_H^\pi}{d\ {}_H^w} + v_L q_L\, c''_\pi \;\frac{z_L^\pi}{\frac{M(\theta)}{\theta}\ {}_L} $$ [redacted]

$$ + \ (\ \ {}_k\ \ l) M(\ )\left[\frac{{}_k\left(\frac{l_k}{M(\theta)\, w_k}\frac{dz_k^w}{d\tau_H^w}\right.}{{}_k\ \ l} + \frac{M'(\ )}{M(\ )} \frac{m}{\ } \right. $$ [redacted] $$ \frac{\frac{l_k}{M(\theta)\, w_k}\frac{dz_k^w}{d\tau_H^w}}{{}_k\ \ l} \Bigg] (a+b) = $$

[redacted]

$$ = \ {}_H l_H\, c''_w \;\frac{z_H^w}{M(\ )\, w_H} \quad \frac{1}{M(\ )\, w_H}\frac{dz_H^w}{d\ {}_H^w}\frac{z_H^w}{z_H^w}\frac{M(\ )\,(}{c'_w(z_H^w =} $$ [redacted]

$$ + \ {}_L l_L\, c''_w \;\frac{z_L^w}{M(\ )\, w_L} \quad \frac{1}{M(\ )\, w_L}\frac{dz_L^w}{d\ {}_H^w}\frac{z_L^w}{z_L^w}\frac{M(\ )\,(1 - \ {}_L^w) w_L}{c'_w(z_L^w = M(\ )\, w_L)} $$

$$ +v_H q_H\, c''_\pi \;\frac{z_H^\pi}{\frac{M(\theta)}{\theta}\ {}_H} \quad \frac{1}{\frac{M(\theta)}{\theta}\ {}_H}\frac{dz_H^\pi}{d\ {}_H^w}\frac{z_H^\pi}{z_H^\pi}\frac{M(\ )}{\ }\frac{(1 - \ {}_H^\pi)\ {}_H}{c'_\pi(z_H^\pi = \ {}_H)} $$

$$ +v_L q_L\, c''_\pi \;\frac{z_L^\pi}{\frac{M(\theta)}{\theta}\ {}_L} \quad \frac{1}{\frac{M(\theta)}{\theta}\ {}_L}\frac{dz_L^\pi}{d\ {}_H^w}\frac{z_L^\pi}{z_L^\pi}\frac{M(\ )}{\ }\frac{(1 - \ {}_L^\pi)\ {}_L}{c'_\pi\left(z_L^\pi = \frac{M(\theta)}{\theta}\ {}_L\right.} $$

$$ + \ (\ \ {}_k\ \ l) M(\ )\left[\ {}_k\left(\frac{l_k}{M(\theta)\, w\mathrm{wH}}\right. \right. $$

$$= \quad {}_H l_H \frac{1}{''^w_H} \frac{dz^w_H}{d \;\; ^w_H} \frac{1}{z^w_H}$$

$_{H}^{\pi}$, $_{L}^{\pi}$

$$(\Delta_1 + \Delta_2 - 1)\quad (1 - \quad)^{1 - \quad {}_{L}^{w}}$$

F

$$
= 1 - \frac{\left[\begin{array}{l}\left(\frac{\delta_H l_H}{{}_k\,\delta l}\frac{v_L q_L}{{}_m\,vq} + \frac{\delta_L l_L}{{}_k\,\delta l}\frac{v_H q_H}{{}_m\,vq}\right. w_{HH} + \left(\frac{\delta_H l_H}{{}_k\,\delta l}\frac{v_H q_H}{{}_m\,vq} + \frac{\delta_L l_L}{{}_k\,\delta l}\frac{v_L q_L}{{}_m\,vq}\right. w_{HL} \quad {}^{w}_{H} \\ + \left(\frac{\delta_H l_H}{{}_k\,\delta l}\frac{v_H q_H}{{}_m\,vq} + \frac{\delta_L l_L}{{}_k\,\delta l}\frac{v_L q_L}{{}_m\,vq}\right. w_{LH} + \left(\frac{\delta_H l_H}{{}_k\,\delta l}\frac{v_L q_L}{{}_m\,vq} + \frac{\delta_L l_L}{{}_k\,\delta l}\frac{v_H q_H}{{}_m\,vq}\right. w_{LL} \quad {}^{w}_{L} \\ + \left(\frac{\delta_H l_H}{{}_k\,\delta l}\frac{v_L q_L}{{}_m\,vq} + \frac{\delta_L l_L}{{}_k\,\delta l}\frac{v_H q_H}{{}_m\,vq}\right. {}_{HH} + \left(\frac{\delta_H l_H}{{}_k\,\delta l}\frac{v_H q_H}{{}_m\,vq} + \frac{\delta_L l_L}{{}_k\,\delta l}\frac{v_L q_L}{{}_m\,vq}\right. {}_{HL} \quad {}^{\pi}_{H} \\ + \left(\frac{\delta_H l_H}{{}_k\,\delta l}\frac{v_H q_H}{{}_m\,vq} + \frac{\delta_L l_L}{{}_k\,\delta l}\frac{v_L q_L}{{}_m\,vq}\right. {}_{LH} + \left(\frac{\delta_H l_H}{{}_k\,\delta l}\frac{v_L q_L}{{}_m\,vq} + \frac{\delta_L l_L}{{}_k\,\delta l}\frac{v_H q_H}{{}_m\,vq}\right. {}_{LL} \quad {}^{\pi}_{L}\end{array}\right]}{\frac{1-\tau_H^w}{\varepsilon_H^w} w_H + \frac{1-\tau_L^w}{\varepsilon_L^w} w_L + \frac{1-\tau_H^\pi}{\varepsilon_H^\pi} \quad {}_H + \frac{1-\tau_L^\pi}{\varepsilon_L^\pi} \quad {}_L} \; ^{-1};
$$

$$
= \left(1 - \frac{E_{(s)} w_{Hs} \; {}^{w}_{H} + E_{(s)} w_{Ls} \; {}^{w}_{L} + E_{(s)} \; {}_{sH} \; {}^{\pi}_{H} + E_{(s)} \; {}_{sL} \; {}^{\pi}_{L}}{\frac{1-\tau_H^w}{\varepsilon_H^w} w_H + \frac{1-\tau_L^w}{\varepsilon_L^w} w_L + \frac{1-\tau_H^\pi}{\varepsilon_H^\pi} \; {}_H + \frac{1-\tau_L^\pi}{\varepsilon_L^\pi} \; {}_L}\right)^{-1}; \qquad (64)
$$

s $\left(\frac{\delta_H l_H}{{}_k\,\delta l}\frac{v_L q_L}{{}_m\,vq} + \right.$

c'

$$'' = \frac{1}{\;}=\frac{c''}{c'} = \frac{1}{\;-1}:$$

, A, :

.

2 $c = A(\;^{\gamma} + \;^{\beta})$ ≥ 3, $<$, $c' = A(\;^{\gamma-1} + \;^{\beta-1})$,

$$c'' = A(\;(\;-1)\;^{\gamma-2} + \;(\;-1)\;^{\beta-2}) > 0;$$

$$'' = \frac{\;^{\gamma-2} + \;^{\beta-2}}{(\;-1)\;^{\gamma-2} + (\;-1)\;^{\beta-2}}:$$

A , , .

$$\frac{@''}{@} = -\;^{\beta+\gamma-5}(\;-\;)^2 < 0:$$

N ≥ 3 $= \;-1$. . F , < 1 $= 3$ $= 2$, > 1 $= 2$ $= 1$.

$\frac{1-\tau_H^w}{\varepsilon_H^w} w_H < \frac{1-\tau_L^w}{\varepsilon_L^w} w_L$, (66) -. H $(1-\;_H^w)w_H < (1-\;_L^w)w_L$ $_H^w > \;_L^w$ (66) $_H^w w_H > \;_L^w w_L$. N $(E_{(m)}\;_{Hm}\;_m^\pi - E_{(m)}\;_{Lm}\;_m^\pi)$. - $_{HH} > \;_{LH}$ $_{HL} > \;_{LL}$ $(E_{(m)}\;_{Hm}\;_m^\pi - E_{(m)}\;_{Lm}\;_m^\pi > 0)$. H (66) . $(1-\;_H^w)w_H > (1-\;_L^w)w_L$, $_H > \;_L$, $''^w_H \leq ''^w_L$. □

Proof of Proposition 11.

fi (65). fi . , (60)-(63) . fi , . D

(60) (61)

$$(\Delta_1 + \Delta_2 - 1) \quad (1 - \quad) \frac{1 - \quad ^{w}}{\quad ''_{w}} w + \quad (w \quad ^{w} + \quad ^{\pi} - (1 - \quad) \bar{R}) \quad =$$

$$= (1 - \quad) \left[(1 - \quad ^{w}) w + \quad (w \quad ^{w\,''w} + \quad ^{\pi\,''w} - (1 - \quad) \bar{R}^{\,''w}) \right]$$

$$-$$

(68), (69), P 11.

P 12

(1 −), (1 −)= ↑,

$^{\pi}=$ ^{w} ↓.□