Land Taxation in a Dualcentric City

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Abstract

While there is a large literature both on the spatial impacts of taxation and competing land uses in a dualcentric city, the two topics have yet to be analyzed under a common framework. This paper describes the optimal choice of a differential land tax across competing land uses in the framework of a dualcentric city model. The key results of the analysis indicate that higher levels of spatial competition lead to smaller tax differentials. These differentials are also sensitive to the relative size of the tax bases and the slopes of the bid rent functions at the respective land-use boundaries.

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to illustrate the impact of state imposed differentials on different jurisdiction types.

2 Background

The literature on the spatial decisions of firms and households in a dualcentric city has focused on two main areas; one in which households purchase goods from firms, and the other where there is a link between workers and employment centers. The endogenous formation of this city type was first examined in the framework of Fujita and Thisse (1986), where the Hotelling (1929) linear market model was combined with land consumption in the tradition of VonThunen (1826) and Alonso (1964). In this model, household location decisions are based on firm locations, and spatial competition is examined when the households simultaneously choose their consumption levels of land as well as the firm's output. In this early work, it was assumed households have constant population densities over locations. In a similar line of research, Fujita and Thisse (1991) relax this constant population density assumption to enable the identification of differing types of land use outcomes. Here two different firms choose their locations under the assumption of a spatial duopoly. After the firms choose their best locations, households are free to locate within the jurisdiction [1]. By relaxing this fixed population density constraint, the authors are able to characterize three different land use outcomes in this spatial duopoly model: i) a monocentric city in which firms locate at the center - occurs in small land areas, ii) a dualcentric city where firms locate separately and residents directly compete for land, and finally iii) a city where firms and residential patterns are completely separated - occurs in jurisdictions with large areas. These differing land use outcomes are used in the evaluation of the tax structure examined in this paper.

[1] A key assumption in these models is that households are making their choice in an open city framework. Therefore, the jurisdiction may be thought of as being located in a system of cities, which results in a fixed level of household utility, exogenously defined within the system. Therefore, at any chosen location, the household will be indifferent between the current choice and other choices which satisfy the utility constraint.

A second source of endogeneity in the locational choice of households is through spatially delineated labor markets. Examples of this are described in Smith (1997), Gabszewicz and Thisse (1986), and Fujita et al. (1997). Whatever the modeling strategy used, the existence of the endogenous formation of multiple centers in a city has been extensively explored in the literature. In the current paper, the location of the city centers are taken as given, and the choice of the optimal land tax under differing land use assumptions is examined.

The analysis of taxation in a spatial framework has been extensively studied in the monocentric city framework, with differing assumptions regarding the disposal of tax revenues and the closed vs. open nature of the city. These models are traditionally based on the spatial, monocentric city model of Wheaton (1974). This analysis shows how composition, the urban boundary, and lot sizes change in response to perturbations in exogenous variables such as income, reservation utility, location, and travel cost. Wheaton makes the assumption of a utility function which requires only the numerarie and housing goods to be normal, with positive income effects. The utility function lends itself to a comparative static analysis, generalizable to a large set of functional form assumptions. This generalized functional form is used in the current paper. Fujita (1989) describes a similar mechanism in the monocentric city with inclusion of property tax rates which are passed on to an absentee landowner. In these spatial taxation models, tax revenue may be either be freely disposed of, returned as income, or used for the provision of public goods. Grieson (1974), LeRoy (1976) and Carlton (1981) assume the free disposal of tax revenue and examine the effects of the spatial distribution of residents. Brueckner (2003) adopts this approach in his study of the relationship between property taxation and urban sprawl. Polinsky and Rubinfeld (1978) examine the long-run effects of residential property taxes and local public services in an open city urban spatial model. They allow for the adjustment of wages and land prices in response to changes in the local fiscal structure. In this model, the jurisdiction does not have to have a balanced budget, but faces a fixed public good expenditure level. Tax revenues above or

The bid rent function for agents associated with city center one is expressed as

$$\psi(r, u_h) = \max_{z_h, x_h} \left\{ \frac{y_h - T_h(r) - z_h}{x_h} | U(z_h, x_h) = u_h \right\} \quad (1)$$

This is the traditional bid rent construction for a monocentric city with a city center located at $r = 0$.[2] It is costly to travel to the city center, so the bid rent function is a decreasing function of the distance from $r = 0$.

$$\frac{\partial \psi}{\partial r} < 0 \quad (2)$$

The distance between city center one and city center two is normalized to one ($r \in [0, 1]$) without loss of generality.

The bid rent function for city center two agents may then be defined as

$$\varphi(r, u_s) = max_{z_s, x_s} \left\{ \frac{y_s - T_s(1 - r) - z_s}{x_s} | U(z_s, x_s) = u_s \right\}. \quad (3)$$

The travel cost for a city center two agent at location r is $T(1 - r)$.

$$\frac{\partial \varphi}{\partial r} > 0 \quad (4)$$

Rents are increasing as the distance to city center two is decreased. In the tradition of vonThunen, the price floor of land is the agricultural rent, denoted here by R^a. Agricultural land use may be present in the jurisdiction if all land is not occupied by either of the city center land use types.

It is assumed that the two agent types are homogeneous within group and heterogeneous across groups, so that the values of z, x, y and U for the two types are not necessarily identical. Heterogeneity across groups allows for di�ering bid rent curves between city center one and

[2]Following the assumptions of Wheaton (1974), utility is strictly quasiconcave and both goods x and z have positive income e�ects and are assumed to be normal goods

two border net rents are described as

$$(1-\tau^h)\psi(r^c) = (1-\tau^s)\varphi(r^c) \tag{5}$$

Where r^c is the common boundary shared by the two types. Additionally, at the boundary, the difference between the city center one and city center two bid rent functions is

$$\Delta R^{hs} = \frac{\tau^s - \tau^h}{1-\tau^s}$$

This outcome is presented graphically in the left panel of Figure 2 where the boundary is denoted r^c.

At $\overline{\pi} = \pi(I)$, the increased level of tax revenue required by the jurisdiciton causes a discrete tax policy change to Case II, which describes a spatial structure where the usage types from city center one and city center two net boundary rents are equal to agricultural rent, but no agricultural land exists in the jurisdiction. The main difference associated with Case II is that the price floor for the two usage types at the boundary is constrained by agricultural land, which changes the optimal tax choice of the planner.

If the tax constraint is increased to $\pi(II)$, then the jurisidiction faces another regime switch into Case III. Here, agricultural use outbids land use one and land use two types over some portion of the jurisdiction, and therefore there is not direct competition between the respective city center boudaries. This results in two boundaries; city center one/agricies;J0-23.908ty center

The gross rent differential between city center one and agricultural land is

$$\Delta R^{ha} = \tau^h \psi(r^h)$$

Similarly, the gross border rent differential between agricultural land and city center two is

$$\Delta R^{as} = \tau^s \varphi(r^s)$$

This equilibrium type is presented graphically in the right panel of Figure 2. The City center one land use type occupies the land between $r = 0$ and $r = r^h$, agricultural land is located between r^h and r^s, and city center two occupies the land from r^s to 1. So, for Case III, it is necessary that $r^s > r^h$

These cases differ both in their level of land competition and tax revenue constraints which in turn define the optimal tax differential.

Finally, $\pi(III)$, occurs when one of the land use types reaches a corner solution, as a result of the Laffer effect [5]. Case IV then simply solves the case where tax revenues for one type have been maximized, and the second tax rate is simply set by substituting the corner solution tax rate into the tax revenue constraint. $\pi(max)$ describes the case where both types are at the corner solution. This is the maximum amount of tax revenue which may be raised by the jurisdiction. The following analysis will focus on Cases I-III, as the results for the Case IV restrict the planner's choice of tax instruments, and are simply corner solutions.

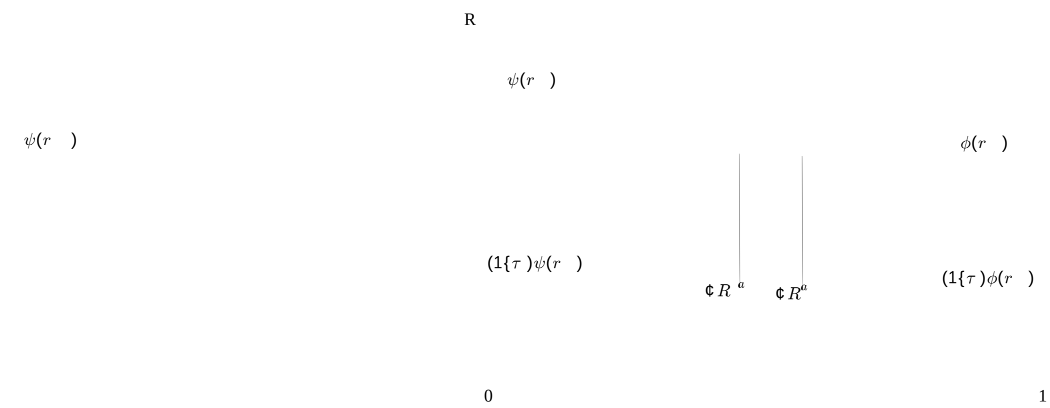
The goal of the central planner is to maximize net rent revenue (NR) subject to a tax revenue constraint, $\bar{\pi}$. The differing sets of the binding constraints on the tax revenue constraint affect the planner's choice of tax levels. It is assumed that the central planner has full control over the ad valorem land taxes levied on the city center one and city center

[5] This effect is discussed in Appendix C

two land use types. The maximization problem is described as

$$max_{\tau^h,\tau^s} \quad NR = (1-\tau^h)^{r^h(\tau^h}$$

Figure 2: Land Use for Case I and Case III



In the three sections that follow, the optimal tax structure under each of these three cases is described. The relative net rent revenue maximizing levels of τ^h and τ^s are shown to be dependent upon the size of the relative tax bases, slopes of the bid rent functions at the boundary and the level of land competition.

4 Rent Maximization for Case I

If the tax revenue constraint can be filled in Case I, then this is the best tax regime[6]. In the following analysis, it is shown that the optimal taxation scheme under direct land competition between the two usage types incorporates identical tax rates for both. For the intuition behind this result, consider a case where tax rates are equal and then city center one tax rates are increased a marginal amount. In order for the tax revenue constraint to hold, city center two tax rates must be decreased. This change causes land use to switch from usage one to usage two at the border, an effect which will strictly decrease rent revenue. This is first shown analytically and then graphically below.

Net rent revenues for the Case I are defined as

$$NR = (1-\tau^h)\int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr + (1-\tau^s)\int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr \tag{15}$$

Where (12) is the only binding constraint. The net revenue maximizing condition is then:[7]

$$\left(\int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr\frac{\partial r^c}{\partial \tau^h} - \int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr\frac{\partial r^c}{\partial \tau^s}\right)\left(\tau^h\psi(r^c) - \tau^s\varphi(r^c)\right) = 0 \tag{16}$$

Proposition 4.1. *In Case I, the optimal tax rates are such that* $\tau_h = \tau_s$.

Proof. Let the solution, (τ_h, τ_s), satisfy the tax revenue constraint. In order for (16) to hold, at least one of the two left hand terms must equal zero.

Step 1: Show that the first term in (16) $\left(\int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr\frac{\partial r^c}{\partial \tau^h} - \int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr\frac{\partial r^c}{\partial \tau^s}\right) \neq 0$.

First note that from the boundary constraint (9).

$$\frac{\partial r^c}{\partial \tau^h} = \frac{\psi(r^c)}{(1-\tau^h)\frac{\partial\psi}{\partial r} - (1-\tau^s)\frac{\partial\varphi}{\partial r}} \tag{17}$$

$$\frac{\partial r^c}{\partial \tau^s} = -\frac{\varphi(r^c)}{(1-\tau^h)\frac{\partial\psi}{\partial r} - (1-\tau^s)\frac{\partial\varphi}{\partial r}} \tag{18}$$

Substituting (17) and (18) into the first term of (16) and setting this equal to 0 yields:

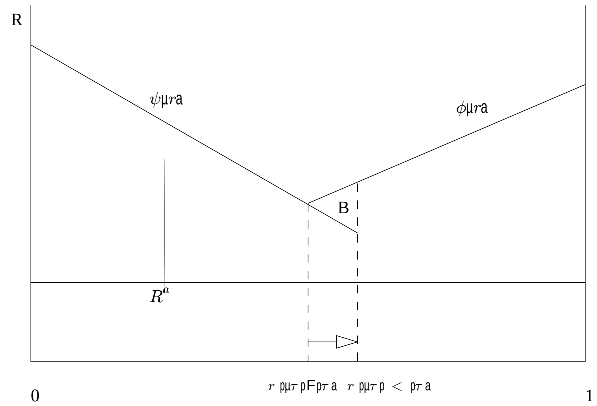
$$\frac{\int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr}{\int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr} = -\frac{\varphi(r^c)}{\psi(r^c)}$$

Which cannot hold if $\{\int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr,\ \int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr,\ \psi(r^c),\ \varphi(r^c)\} > 0$.

Thus, for the first order condition to hold, $\tau^h\psi(r^c) - \tau^s\varphi(r^c)$ must equal 0. This condition is satisfied when $\tau_h = \tau_s$. Therefore, under Case I, the rent revenue maximizing tax pair

[7] Derivation is shown in Appendix A

Figure 3: Case I: Tax Pairs



(τ_h, τ_s) is such that $\tau_h = \tau_s$. □

The intuition of this result is clear from Figure 3. Any differential between the tax rates shifts the boundary relative to the case where $\tau^h = \tau^s$ and results in an unambiguous loss in rent revenue, shown as Area B in the figure (for the case $\tau^s > \tau^h$). The distortionary aspect of this differential tax on the spatial distribution of differing land use types causes a decrease in the rent envelope. This equilibrium type corresponds to a jurisdiction with relatively high rents and population densities, where land is valued significantly higher than agricultural use over all locations. In this case, the uniform tax rates used to generate the tax revenues do not cause any distortions in the land market. This is not the result in the Case II or Case III land distribution types, as interactions with agricultural land cause the planner to implement a differential tax between types which will distort land use outcomes.

The upper bound on tax revenue that can be raised in Case I is denoted as $\overline{\pi}_1$, and defines the transition between Cases I and II. This is described as:

$$\bar{\pi}_1 = max\left(\tau \int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr + \tau \int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr\right)$$

Subject to

$$r^h = r^s = r^c$$

$$\psi(r^c) = \varphi(r^c)$$

$$\tau^h = \tau^s = \tau$$

In cases where $\bar{\pi} > \bar{\pi}_1$, the planner is forced to set tax rates higher than is possible under equal tax rates. The analysis of these possible tax regimes is described next.

5 Rent Maximization for Case II

In Case II, the planner sets the relative tax rates so that there is still not any agricultural land in the jurisdiction, but the net rents for each usage type equal the agricultural rents at the unique boundary. The transition from Case I to Case II occurs as the tax revenue requirement increases to the point where the tax rates make the landowners indifferent between agricultural and other usage types at the boundary.

The landowners then receive total net rent revenue equal to

$$NR = (1-\tau^h) \int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr + (1-\tau^s) \int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr \qquad (19)$$

Where constraints (10), (11), and (12) are binding. The tax revenue constraint is described by:

$$\overline{\pi} = \tau^h \int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr + \tau^s \int_{r^c(\tau^h,\tau^s)}^1 \varphi(r)dr \tag{20}$$

The solution to the planner's problem is then [8].

$$\frac{\int_{r^c(\tau^h,\tau^s)}^1 \varphi(r)dr}{-\frac{\varphi(r^c)}{(1-\tau^s)\frac{\partial\varphi}{\partial r}}} = \frac{\int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr}{\frac{\psi(r^c)}{(1-\tau^h)\frac{\partial\psi}{\partial r}}} \tag{21}$$

At the optimal tax solution (τ_h, τ_s), the marginal cost in terms of rent revenue of increasing tax revenues from either usage must be equal. Here, the absolute loss in rent revenue from increasing τ^s is given by $-\frac{\varphi(r^c)}{(1-\tau^s)\frac{\partial\varphi}{\partial r}} < 0$, and the loss from increasing τ^h is $\frac{\varphi(r^c)}{(1-\tau^h)\frac{\partial\psi}{\partial r}} < 0$. These loses represent the effect of land use switching between land use type one and two. Under most specifications of the total rents and bid rent slopes, this will lead to a case where the optimal tax differential is $\tau^h \neq \tau^s$.

In order to examine this result in more detail, the optimal tax pairs under differing assumptions on the relative bid rent slopes at the land use borders $(\frac{\partial\psi}{\partial r}, \frac{\partial\varphi}{\partial r})$, as well as relative gross rents for each type ($\int_0^{r^c(\tau^h,\tau^s)} \psi($ [illegible]

$$\frac{\int_{r^c(\tau^h,\tau^s)}^{1}\varphi(r)dr}{\int_0^{r^c(\tau^h,\tau^s)}\psi(r)dr} = \frac{-\frac{\varphi(r^c)}{(1-\tau^s)\bar{m}}}{\frac{\psi(r^c)}{(1-\tau^h)\bar{m}}} \tag{22}$$

Step 1: Prove by contradiction that $\tau_h \neq \tau_s$ at the optimum.

Assume $\tau^h = \tau^s$, then (22) becomes

Reducing this expression yields

$$\frac{\int_{r^c(\tau^h,\tau^s)}^{1}\varphi(r)dr}{\int_0^{r^c(\tau^h,\tau^s)}\psi(r)dr} = 1$$

However, since $\frac{\int_{r^c(\tau^h,\tau^s)}^{1}\varphi(r)dr}{\int_0^{r^c(\tau^h,\tau^s)}\psi(r)dr} \neq 1$, this equality does not hold.

Step 2: Show that if $\int_{r^c(\tau^h,\tau^s)}^{1}\varphi(r)dr > \int_0^{r^c(\tau^h,\tau^s)}\psi(r)dr$ then $\tau_s > \tau_h$

Rewrite (22) as

$$\frac{(1-\tau^h)}{\psi(r^c)}\left(\int_0^{r^h(\tau^h)}\psi(r)dr\right) - \frac{(1-\tau^s)}{\varphi(r^c)}\left(\int_{r^s(\tau^s)}^{1}\varphi(r)dr\right) = 0 \tag{23}$$

However, if $\tau^h = \tau^s$,

$$\frac{(1-\tau^h)}{\psi(r^c)}\left(\int_0^{r^h(\tau^h)}\psi(r)dr\right) - \frac{(1-\tau^s)}{\varphi(r^c)}\left(\int_{r^s(\tau^s)}^{1}\varphi(r)dr\right) < 0 \tag{24}$$

Therefore, it is necessary to change the relative tax rates in order for (24) to hold. In order for a rise in τ^s to move toward the rent maximum, the first term must be increasing in τ^s and the second term must be decreasing.

Starting with the second term, let $g(\tau^s) = (1-\tau^s)$ then $\frac{\partial g(\tau^s)}{\partial \tau^s} < 0$. Additionally, $\frac{\partial \varphi(r^c)^{-1}}{\partial \tau^s} < 0$ Since

$\partial\varphi(r^c)$

as $\frac{\partial \varphi}{\partial r} > 0$ and $\frac{\partial r^c}{\partial \tau^s} > 0$. Finally, $\frac{\partial \int_{r^s(\tau^s)}^{1} \varphi(r)dr}{\partial \tau^s} = -\varphi(r^s)\left(\frac{\partial r^s}{\partial \tau^s}\right) < 0$. Therefore, the right side of this term in decreasing in τ^s.

Next, for the first term, let $g(\tau^h(\tau^s)) = (1 - \tau^h)$ and $\frac{\partial g}{\partial \tau^s} > 0$ since $\frac{d\tau^h}{d\tau^s} < 0$ [9]. Additionally, $\frac{\partial \psi^{-1}}{\partial \tau^s} = -\psi(r^c)^{-2}\frac{\partial \psi}{\partial r}\frac{\partial r^c}{\partial \tau^h}\frac{d\tau^h}{d\tau^s} > 0$ since $\frac{\partial \psi}{\partial r} < 0$, $\frac{\partial r^c}{\partial \tau^h} < 0$, $\frac{d\tau^h}{d\tau^s} < 0$. Finally, the term $\frac{\partial \int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr}{\partial \tau^s} = \psi(r^h)\left(\frac{\partial r^c}{\partial \tau^h}\frac{d\tau^h}{d\tau^s}\right) > 0$, so the left hand side is increasing in τ^s, and the condition for the optimal pair of (τ_s, τ_h) is $\tau_s > \tau_h$.

Step 3: If $\int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr > \int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr$ then $\tau_h > \tau_s$

Follows from Step 2. □

Thus, in general under Case II, the optimal tax strategy involves shifting the tax burden toward the larger tax base. Next, I consider a case where, under equal tax rates, the size of the tax bases are equal but the slope of the rent curves at the boundary di er.

Proposition 5.2. *In Case II, when the slopes of the rent curves di er at the boundaries but the tax bases are the equal ($\int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr = \int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr$, $\frac{\partial \varphi}{\partial r} \neq -\frac{\partial \psi}{\partial r}$) under equal tax rates, the optimal tax di erential between types is dependent upon the relative magnitude of the slopes. The optimal tax pair is defined as (τ_h, τ_s) where $\tau_s > \tau_h$ if $\frac{\partial \varphi}{\partial r} > -\frac{\partial \psi}{\partial r}$ and $\tau_h > \tau_s$ if $\frac{\partial \varphi}{\partial r} < -\frac{\partial \psi}{\partial r}$.*

Let $\int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr = \int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr = \bar{k} > 0$

Proof. Step 1: Prove by contradiction that $\tau_h \neq \tau_s$.

Assume that $\tau^h = \tau^s$. Then (21) becomes

$$-\frac{\partial \psi}{\partial r} = \frac{\partial \varphi}{\partial r}$$

However, since $\frac{\partial \psi}{\partial r} \neq -\frac{\partial \varphi}{\partial r}$, $\tau_s = \tau_h$ cannot be the solution.

[9]The sign of $\frac{d\tau^h}{d\tau^s}$ is found by totally di erentiating the tax revenue constraint, and showing that this sign may only be negative at the rent revenue maximizing levels of $(\tau_h^*, \tau_s^*$, see Appendix C for a proof

Step 2: Prove that if $\frac{\partial \varphi}{\partial r} > -\frac{\partial \psi}{\partial r}$ then $\tau_s > \tau_h$.

Let $\bar{m} = \frac{\partial \varphi}{\partial r} > -\frac{\partial \psi}{\partial r} = -\beta\bar{m}$ where $\beta \in (0, 1)$.

Substituting these conditions into 21 yields:

$$\frac{(1-\tau^s)}{\varphi(r^c)} - \frac{\beta(1-\tau^h)}{\psi(r^c)} = 0$$

Using the identities for Case II with di�ering total rents, the condition for that optimal pair, (τ_s, τ_h) is $\tau_s > \tau_h$.

Step 3: If $-\frac{\partial \psi}{\partial r} > \frac{\partial \varphi}{\partial r}$ then $\tau_h > \tau_s$.

The proof of this result follows Step 2. □

Thus, in general under Case II, the optimal tax strategy involves shifting the tax burden toward the usage type with the steeper bid rent curve.

Again, the choice of (τ_h, τ_s) is dependent upon generating enough tax revenue to satisfy the tax revenue constraint. In Case II, the maximum amount of tax revenue which can be generated is given by $\bar{\pi}_2$.

$$\bar{\pi}_2 = max\left(\tau_h \int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr + \tau_s \int_{r^c(\tau^h,\tau^s)}^1 \varphi(r)dr\right)$$

Subject to

$$r^h = r^s = r^c$$

$$(1-\tau_h)\psi(r^c) = (1-\tau_s)\varphi(r^c)$$

$$(\tau_h, \tau_s) \text{ satisfies } \frac{\int_{r^c(\tau^h,\tau^s)}^1 \varphi(r)dr}{\int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr} = \frac{\frac{\varphi(r^c)}{(1-\tau^s)\bar{m}}}{\frac{\psi(r^c)}{(1-\tau^h)\bar{m}}}$$

6 Rent Maximization for Case III

If the planner is still not able to generate enough revenue, then tax rates must be set a such a level that induces a land use type defined by Case III. Here, there is not direct competition for land between the city center one and city center two land usage types. As a result, the only connection between the two types is through the tax revenue constraint. For a simple example of this, consider a case where, starting from identical tax rates that meet the tax revenue constraint, the city center one tax is increased a marginal amount. This increase in the tax rate causes a strict decrease in net rents at all locations covered by the city center one land-use type. This drop results in land use switching as agricultural outbids the city center one land use at the city center one/agricultural border. The increase in tax revenues allows for a corresponding decrease in city center two tax rates, thereby expanding city center two. This results in impacts on net rent revenues which are not obvious, as the gain in rents from the expansion of city center two may or may not equal the loss in revenues from the

The solution to the planner's problem is then [10].

$$\frac{\int_{r^s(\tau^s)}^{1}\varphi(r)dr}{-\tau^s\varphi(r^s)\frac{\partial r^s}{\partial \tau^s}} = \frac{\int_0^{r^h(\tau^h)}\psi(r)dr}{\tau^h\psi(r^h)\frac{\partial r^h}{\partial \tau^h}} = \tag{27}$$

$\frac{\partial r^h}{\partial \tau^h}$ is found by di□erentiating Equation 10 and using the result that $\frac{\partial \psi}{\partial r} < 0$ from (2). So,

$$\frac{\partial r^h}{\partial \tau^h} = \quad \psi(r^h$$

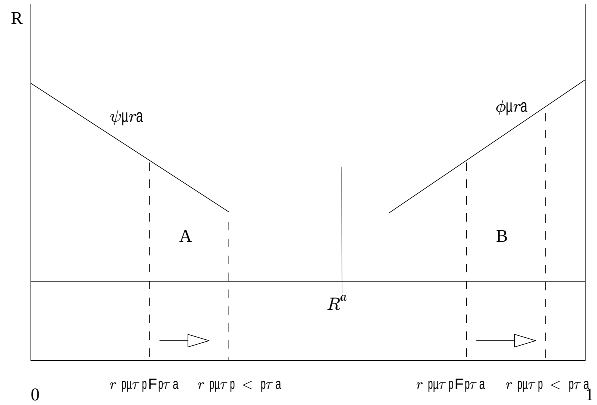


Figure 4: Optimal Tax Differential for Case III with Symmetric Bid Rent Functions

Under Case III, the optimal tax policy for symmetric bid rent functions when the tax rates are equal is $(\tau_h = \tau_s)$. This result is shown graphically in Figure 3[11]. Any movement away from this tax combination results in strictly lower rent revenues, since, from the figure, the gain from A is outweighed by the loss in B.

If the bid rent curves are not symmetric when the tax rates are equal, then the optimal

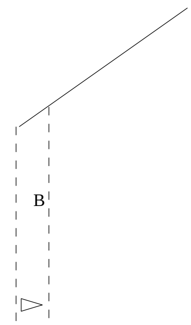


Figure 5: Case III: Tax Pairs for Non-Symmetric Tax Base $\left(\int_{r^s(\tau^s)}^{1} \varphi(r)dr > \int_{0}^{r^h(\tau^h)} \psi(r)dr\right)$

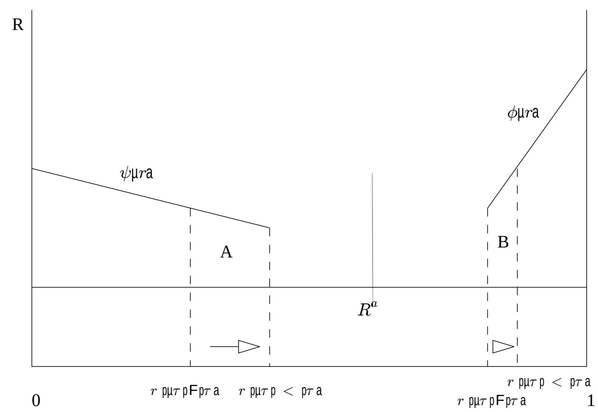


Figure 6: Case III: Tax Pairs in Non-Symmetric boundary bid-rent curves ($\frac{\partial \varphi}{\partial r} > -\frac{\partial \psi}{\partial r}$)

The final analysis for Case III is where, under equal tax rates, the slope of city center one and city center two rent curves di er at the boundary, but the total gross rents from each side are equal. Again, a di erential tax is preferred by the planner. The intuition for the case where $\frac{\partial \varphi}{}$

Results from Case III show that in general the only time it is efficient to set $\tau^h = \tau^s$ is under the assumption of completely symmetric bid rent curves under equal tax rates.

away from this tax regime will cause an unambiguous loss in rent revenues. In less densely developed areas, where the price floor is binding, the use of differential tax will give the policy maker the ability to increase revenues. Therefore, if a local jurisdiction has complete control over the tax rates, land use must be a consideration in the decision of how much to tax. In cases where taxes are set at the state level, it is shown that the implementation of a forced tax differential will serve to lower total rents in urban areas, with ambiguous but identifiable results for less densely populated jurisdictions.

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A Case I: Revenue Maximization

Write the Lagrangian as

$$L = (1-\tau^h)\int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr + (1-\tau^s)\int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr + \lambda\left(\tau^h \int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr + \tau^s \int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr\right) \tag{31}$$

The first order conditions for the Lagrangian are

$$\frac{\partial L}{\partial \tau^h} = -\int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr + (1-\tau^h)\psi(r^c)\frac{\partial r^c}{\partial \tau^h} - (1-\tau^s)\varphi(r^c)\frac{\partial r^c}{\partial \tau^h}$$
$$+\lambda\left(\int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr + \tau^h\psi(r^c)\frac{dr^c}{d\tau^s} - \tau^s\varphi(r^c)\frac{\partial r^c}{\partial \tau^h}\right) = 0$$

$$\frac{\partial L}{\partial \tau^s} = -\int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr + (1-\tau^h)\varphi(r^c)\frac{\partial r^c}{\partial \tau^s} - (1-\tau^s)\varphi(r^c)\frac{\partial r^c}{\partial \tau^s}$$
$$+\lambda\left(\tau^h\varphi(r^c)\frac{\partial r^c}{\partial \tau^s} + \int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr - \tau^s \quad {}_1{}^{c})^{\partial r} \quad (\quad + - \quad (\right.$$

C Case II: $\frac{d\tau^h}{d\tau^s}$

The relationship between τ^h and τ^s can be seen by totally di�erentiating the tax revenue constraint, given as

$$\tau^h \int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr + \tau^s \int_{r^c(\tau^h,\tau^s)}^1 \varphi(r)dr = \overline{\pi} \tag{34}$$

and

$$d\tau^h \int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr + \tau^h \frac{\partial r^c}{\partial \tau^h}\psi(r^c)d\tau^h + d\tau^s \int_{r^c(\tau^h,\tau^s)}^1 \varphi(r)dr - \tau^s \frac{\partial r^c}{\partial \tau^s}\varphi(r^h) = 0 \tag{35}$$

Solving for $\frac{d\tau^h}{d\tau^s}$ yields:

$$\frac{d\tau^h}{d\tau^s} = -\frac{-\tau^s \frac{\partial r^c}{\partial \tau^s}\varphi(r^c) + \int_{r^c(\tau^h,\tau^s)}^1 \varphi(r)dr}{\tau^h \frac{\partial r^c}{\partial \tau^h}\psi(r^c) + \int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr} \tag{36}$$

Proposition C.1. *At the rent revenue maximizing levels of* (τ_h, τ_s), $\frac{d\tau^h}{d\tau^s} < 0$

Proof. Proof by contradiction.

Step 1: Assume that $\frac{d\tau^h}{d\tau^s} > 0$. There are two cases where this may hold . The first case where the numerator of (36) is positive and the denominator is negative. The second case is simply the contrapositive of the first.

These conditions are defined as

$$-\tau^s \frac{\partial r^c}{\partial \tau^s}\varphi(r^c) + \int_{r^c(\tau^h,\tau^s)}^1 \varphi(r)dr > 0$$

$$\tau^h \frac{\partial r^c}{\partial \tau^h}\psi(r^c) + \int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr < 0$$

Rearranging terms and combining these conditions yields

$$\frac{\int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr}{-\tau^h \frac{\partial r^c}{\partial \tau^h}\psi(r^c)} < 1 < \frac{\int_{r^c(\tau^h,\tau^s)}^1 \varphi(r)dr}{\tau^s \frac{\partial r^c}{\partial \tau^s}\varphi(r^c)}$$

However, from the solution to the maximization problem,

$$\frac{\int_0^{r^c(\tau^h,\tau^s)} \psi(r)dr}{-\tau^h \frac{\partial r^c}{\partial \tau^h}\psi(r^c)} = \frac{\int_{r^c(\tau^h,\tau^s)}^1 \varphi(r)dr}{\tau^s \frac{\partial r^c}{\partial \tau^s}\varphi(r^c)}$$

The same result applies to the contrapositive, and therefore $\frac{d\tau^h}{d\tau^s} > 0$ does not exist at the maximum.

Step 2: Assume that $\frac{d\tau^h}{d\tau^s} = 0$ Then

$$\tau^s \frac{\partial r^c}{\partial \tau^s}\varphi(r^c) = \int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr$$

Then, from the maximization results, the following condition must hold:

$$-\tau^h \frac{\partial r^c}{\partial \tau^h}\psi(r^c) = \int_{0}^{r^c(\tau^h,\tau^s)} \psi(r)dr$$

However, if this is true, then the denominator of $\frac{d\tau^h}{d\tau^s} = 0$, and the solution is undefined. □

So, if $\frac{d\tau^h}{d\tau^s}$ must be less than zero, there still remain two possible (τ^h, τ^s) pairs which satisfy, which correspond to the the possible equilibrium points, that in which the both the denominator and numerator share the same sign, as a result of a Laffer-type curve. In the first case (τ_l^h, τ_l^s), the numerator and denominator are positive, then an increase in τ^h or τ^s will cause tax revenues to increase, which corresponds to being on the left hand side of the Laffer curve. However, if both are negative (τ_r^h, τ_r^s), then we are to the right of the Laffer curve. In the case when both are negative, the planner may increase both tax and rent revenues by decreasing the either of the tax rates. Therefore, it is clear that if $\tau_l^h < \tau_r^h$, and $\tau_l^s < \tau_r^s$ then

$$NR(\tau_l^h, \tau_l^s) > NR(\tau_r^h, \tau_r^s) \tag{37}$$

since

$$\frac{\partial NR}{\partial \tau^h} = -\int_{0}^{r^c(\tau^h,\tau^s)} \psi(r)dr + (1-\tau^h)\psi(r^c)\frac{\partial r^c}{\partial \tau^h} < 0$$

$$\frac{\partial NR}{\partial \tau^s} = -\int_{r^c(\tau^h,\tau^s)}^{1} \varphi(r)dr - (1-\tau^s)\varphi(r^c)\frac{\partial r^c}{\partial \tau^s} < 0$$

D Case III: Revenue Maximization

The planner's problem is

$$\max_{\tau^h,\tau^s} \quad (1-\tau^h)\int_{0}^{r^h(\tau^h)} \psi(r)dr + \int_{r^h(\tau^h)}^{r^s(\tau^s)} R^a dr + (1-\tau^s)\int_{r^s(\tau^s)}^{1} \varphi(r)dr$$

$$\text{s.t.} \quad \tau^h \int_{0}^{r^h(\tau^h)} \psi(r)dr + \tau^s \int_{r^s(\tau^s)}^{1} \varphi(r)dr = \overline{\pi}$$

The Lagrangian is then defined as

$$L = (1-\tau^h) \int_0^{r^h(\tau^h)} \psi(r)dr + \int_{r^h(\tau^h)}^{r^s(\tau^s)} R^a dr + (1-\tau^s) \int_{r^s(\tau^s)}^{1} \varphi(r)dr + \lambda \left(\tau^h \int_0^{r^h(\tau^h)} \psi(r)dr + \tau^s \int_{r^s(\tau^s)}^{1} \varphi(r)dr \right)$$

7 The first order conditions are then

$$\frac{\partial L}{\partial \tau^h} = (1-\tau^h)\psi(r^h)\frac{\partial r^h}{\partial \tau^h} - \int_0^{r^h(\tau^h)} \psi(r)dr - R^a \frac{\partial r^h}{\partial \tau^h} + \lambda \left(\tau^h \psi(r^h) \frac{\partial r^h}{\partial \tau^h} + \int_0^{r^h(\tau^h)} \psi(r)dr \right) = 0 \quad (38)$$

$$\frac{\partial L}{\partial \tau^s} = R^a \frac{\partial r^s}{\partial \tau^s} - (1-\tau^s)\varphi(r^s)\frac{\partial r^s}{\partial \tau^s} - \int_{r^s(\tau^s)}^{1} \varphi(r)dr + \lambda \left(-\tau^s \varphi(r^s) \frac{\partial r^s}{\partial \tau^s} + \int_{r^s(\tau^s)}^{1} \varphi(r)dr \right) = 0 \quad (39)$$

Combining (38) and (39) yields

$$\psi(r^h)\frac{\partial r^h}{\partial \tau^h}\tau^s\varphi(r^s)\frac{\partial r^s}{\partial \tau^s} - \psi(r^h)\frac{\partial r^h}{\partial \tau^h}\int_{r^s(\tau^s)}^{1} \varphi(r)dr - R^a\frac{\partial r^h}{\partial \tau^h}\tau^s\varphi(r^s)\frac{\partial r^s}{\partial \tau^s}$$

$$+ R^a\frac{\partial r^h}{\partial \tau^h}\int_{r^s(\tau^s)}^{1} \varphi(r)dr + R^a\tau^h\psi(r^h)\frac{\partial r^h}{\partial \tau^h}\frac{\partial r^s}{\partial \tau^s} + R^a\frac{\partial r^s}{\partial \tau^s}\int_0^{r^h(\tau^h)} \psi(r)dr$$

$$-\tau^h\psi(r^h)\frac{\partial r^h}{\partial \tau^h}\varphi(r^s)\frac{\partial r^s}{\partial \tau^s} - \varphi(r^s)\frac{\partial r^s}{\partial \tau^s}\int_0^{r^h(\tau^h)} \psi(r)dr = 0$$

From the city center one boundary condition - $(1-\tau^h)\psi(r^h) = R^a$

$$\psi(r^h) - R^a = \frac{\tau^h R^a}{(1-\tau^h)}$$

and from the city center two boundary condition - $(1-\tau^s)\varphi(r^s) = R^a$

$$R^a - \varphi(r^s) = -\frac{\tau^s R^a}{(1-\tau^s)}$$

Substituting these identities into 43 yields

$$\frac{\tau^s\tau^h\varphi(r^s)R^a}{(1-\tau^h)} - \frac{\tau^h\tau^s\psi R^a}{(1-\tau^s)} = \tau^h\tau^s R^a\left(\frac{\varphi(r^s)}{(1-\tau^h)} - \frac{\psi(r^h)}{(1-\tau^s)}\right) \tag{44}$$

And, since $R^a = (1-\tau^h)\psi(r^h) = (1-\tau^s)\varphi(r^s)$, The interior of this expression is zero, and term (40) is zero.

Step 2:

Now, from (41) and (42)

$$\frac{\partial r^h}{\partial \tau^h}\left(R^a \int_{r^s(\tau^s)}^{1} \varphi(r)dr - \psi(r^h)\int_{r^s(\tau^s)}^{1} \varphi(r)dr\right) + \frac{\partial r^s}{\partial \tau^s}$$

$$\frac{(1-\tau^h)}{\tau^h\psi(r^h)^2}\left(\int_0^{r^h(\tau^h)}\psi(r)dr\right)-\frac{(1-\tau^s)}{\tau^s\varphi(r^s)^2}\left(\int_{r^s(\tau^s)}^{1}\varphi(r)dr\right) < 0 \tag{49}$$

Therefore, it is necessary to change the relative tax rates in order for (48) to hold. In order for a rise in τ^s to move toward the rent maximum, the first term must be increasing in and the second term must be decreasing in τ^s.

Starting with the second term, let $g(\tau^s) = \frac{(1-\tau^s)}{\tau^s}$ then $\frac{\partial g(\tau^s)}{\partial \tau^s} < 0$. Additionally, $\frac{\partial \varphi(r^s)^{-2}}{\partial \tau^s} < 0$ Since

$$\frac{\partial \varphi(r^s)^{-2}}{\partial \tau^s} = -2\varphi(r^s)\frac{\partial \varphi}{\partial r}\frac{\partial r^s}{\partial \tau^s} < 0$$

as $\frac{\partial \varphi}{\partial r} > 0$ and $\frac{\partial r^s}{}$

Substituting these conditions into 30 yields:

$$\frac{-\frac{(1-\tau^h)\beta m}{\tau^h \psi(r^h)^2}\bar{k}}{\frac{(1-\tau^s)m}{\tau^s \varphi(r^s)^2}\bar{k}} = -1$$

And therefore:

$$m\bar{k}\left(\frac{(1-\tau^s)}{\tau^s \varphi^2} - \frac{(1-\tau^h)}{\tau^h \psi^2}\beta\right) = 0$$

Using the identities for the Case III with di�ering tax bases, the condition for that optimal pair, (τ_s, τ_h) is $\tau_s > \tau_h$.

Step 3: If $-\frac{\partial \psi}{\partial r} > \frac{\partial \varphi}{\partial r}$ then $\tau_h > \tau_s$. The proof of this result follows Step 2. □