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Closing International Real Business Cycle Models with Restricted Financial Markets

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Abstract

Several authors argue that international real business cycle (IRBC) models with incomplete financial markets offer a good explanation of the ranking of cross-country correlations. Unfortunately, this conclusion is suspect, because it is commonly based on an analysis of the near steady state dynamics using a linearized system of equations. The baseline IRBC model with incomplete financial markets does not possess a unique deterministic steady state and, as a result, its linear system of difference equations is not stationary. We show that the explanation of the ranking of cross-country correlations is robust to modifications that ensure a unique steady state and a stationary system of linear difference equations. We find, however, that the modifications affect the quantitative predictions regarding key macroeconomic variables.

JEL Classification Codes: F32, G15.
Keywords: Incomplete markets, stationarity, cross-country correlations, wealth effects.

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1. Introduction

The international real business cycle (IRBC) model with incomplete international financial markets is successful at reconciling predicted business cycle moments with empirical moments. In particular, the IRBC model with trade in a one-period bond driven by shocks that are highly persistent and that do not spill over international boundaries solves the *quantity anomaly*. This anomaly, coined by Backus, Kehoe, and Kydland (1995), refers to the inability of the IRBC model with complete markets to correctly predict that the cross-country correlation of output is larger than the cross-country correlation of consumption. Baxter and Crucini (1995) argue that the IRBC model with incomplete markets solves the quantity anomaly because of an important differential wealth effect. In the complete markets model, a rise in home productivity generates a small increase in wealth at home and a large increase in wealth abroad. This arises because complete international financial markets ensure perfect risk sharing. The result is that home and foreign consumption fluctuations are highly correlated. In the incomplete markets model, however, the rise in home productivity generates a large increase in wealth at home, but only a small increase in wealth abroad. This arises because financial markets do not ensure perfect risk sharing. The result is that home and foreign consumption fluctuations need not be highly correlated.

Unfortunately, these conclusions are suspect because they are generated from an analysis of the model's near steady state dynamics. That is, most studies use a linear approximation method similar to that of King, Plosser, and Rebelo (2002). The method requires

stationary incomplete markets models driven by shocks that are highly persistent and that do not spill over international boundaries solve the quantity anomaly. The models driven by shocks that spill over international boundaries, however, do not solve the quantity anomaly. Second, we find that the business cycle moments and impulse responses generated by the different models differ only when shocks are persistent and do not spill over. Thus, the quantitative predictions differ only when the models solve the quantity anomaly. Third, we find that the debt elastic interest rate model and the quadratic portfolio costs model outperform the other stationary models in the sense that they generate business cycle moments that match the empirical moments more closely. Fourth, we find that baseline and stationary models generate a similar wealth effect, but dissimilar price (wages and interest rate) effects. Finally, we show that the ability to solve the quantity anomaly relies on the ability to change the supply of physical capital, but not much on the ability to change the supply of labor. This occurs because of the price effects, and especially of the interest rate effect.

2. A Statement of the Problem

To illustrate the problem, we construct a two-country, dynamic, general equilibrium model with trade in a homogenous good and in a one-period bond. The model is similar to those in Baxter and Crucini (1995) and Kollmann (1996). In what follows, we only describe the home economy, but the foreign economy is symmetric up to country specific productivity shocks. Foreign country variables are identified by an asterisk.

2.1 The Baseline Incomplete Markets (IM) Model

In the IM model, the home economy is populated by a representative consumer and a representative firm. The consumer's expected lifetime utility is

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t, n_t)\right], \tag{1}$$

where E_t is the conditional expectation operator, c_t is consumption, n_t is employment,

$$u(c_t, n_t) = \left[c_t^{\eta}(1-n_t)^{1-\eta}\right]^{\gamma}/\gamma, \tag{2}$$

$0 < \beta < 1$, $\eta > 0$, and $\gamma \leq 1$.

The consumer's budget constraint is

$$c_t + x_t + q_t^w b_{t+1} = w_t n_t + r_t^k k_t + b_t, \tag{3}$$

where x_t denotes investment, w_t is the wage rate, r_t^k is the rental rate of capital, k_t is the capital stock, b_t is the stock of one-period bond, and q_t^w is the world price of the one-period bond. The capital stock evolves according to

$$k_{t+1} = \phi(x_t/k_t)k_t + (1 - \delta)k_t \tag{4}$$

where

$$\phi(x_t/k_t) = \frac{\omega_1}{1 - 1/\xi}\left(\frac{x_t}{k_t}\right)^{1-1/\xi} + \omega_2, \tag{5}$$

$0 < \delta < 1$ and $\xi > 0$. Also, ω_1 and ω_2 are set so that $\phi(x/k) = \delta$ and $\phi_\delta(x/k) = 1$ in the deterministic steady state, where ϕ_{delta} is the derivative of the function $\phi(\cdot)$ with respect to x/k. The function $\phi(\cdot)$ implies an adjustment cost, and ξ is the elasticity of investment with respect to Tobin's q.

The firm's profits are

$$y_t - w_t n_t - r_t^k k_t, \tag{6}$$

where y_t denotes the firm's output. As is standard, output is produced with the constant return to scale technology

$$y_t = z_t k_t^\alpha n_t^{1-\alpha}, \tag{7}$$

where z_t is the level of total factor productivity and $0 < \alpha < 1$.

The model is closed by the asset market clearing condition

$$b_t + b_t^* = 0. \tag{8}$$

Finally, the stationary stochastic process that drives the level of productivity is

$$\begin{bmatrix} \ln(z_t) \\ \ln(z_t^*) \end{bmatrix} = \begin{bmatrix} \rho & \nu \\ \nu & \rho \end{bmatrix}\begin{bmatrix} \ln(z_{t-1}) \\ \ln(z_{t-1}^*) \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon_t^* \end{bmatrix}, \tag{9}$$

where ρ measures the persistence of productivity shocks and ν measures the degree of international spillovers. The vector $\mathbf{e}_t = (\epsilon_t \quad \epsilon_t^*)'$ contains innovations with covariance matrix

$$\Sigma = \begin{bmatrix} \sigma^2 & \psi \\ \psi & \sigma^2 \end{bmatrix}.$$

The competitive consumer chooses consumption, employment, capital and bond holdings to maximize expected lifetime utility (1) subject to the constraints (3) and (4). The competitive firm hires labor and capital to maximize profits (6) subject to the production technology (7). The set of first-order conditions of the consumer's and firm's problems, as well as the asset market clearing condition form the system of equations that characterizes the symmetric equilibrium. The system includes home and foreign variants of

$$\lambda_t = u_{ct}, \tag{10.1}$$

$$u_{nt} = -\lambda_t(1-\alpha)y_t/n_t, \tag{10.2}$$

$$q_t^w = \beta E_t[\lambda_{t+1}]/\lambda_t, \tag{10.3}$$

$$\frac{\lambda_t}{\phi_{\delta t}} = \beta E_t\left[\lambda_{t+1}\alpha\frac{y_{t+1}}{k_{t+1}} + \frac{\lambda_{t+1}}{\phi_{\delta t+1}}\left(\phi_{t+1} - \phi_{\delta t+1}\frac{x_{t+1}}{k_{t+1}} + 1 - \delta\right)\right], \tag{10.4}$$

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha}, \tag{10.5}$$

$$k_{t+1} = \phi_t k_t + (1-\delta)k_t, \tag{10.6}$$

$$y_t = c_t + x_t + q_t^w b_{t+1} - b_t, \tag{10.7}$$

as well as

$$b_t + b_t^* = 0. \tag{10.8}$$

Here, u_{ct} and u_{nt} are the partial derivatives of $u(c_t, n_t)$ with respect to c_t and n_t, and λ_t is the multiplier associated with the budget constraint (3). Equation (10.1) relates the multiplier λ_t to the marginal utility of consumption. Equation (10.2) equates the marginal production technology. Equation (10.6) is the capital accumulation. Equation (10.7) is the national income identity. Finally, equation (10.8) is the asset market clearing condition.

The system (10) has 15 independent equations. These equations must solve for 7 home variables (y_t, c_t, n_t, x_t, k_t, b_t, and λ_t), 7 foreign variables (y_t^*, c_t^*, n_t^*, x_t^*, k_t^*, b_t^* and λ_t^*), and 1 asset price (q_t^w).

2.2 The Problem

As is standard, the equilibrium system (10) does not possess an analytical solution. Most equations are $q_t^w = \beta E_t[\lambda_{t+1}]/\lambda_t$ and $q_t^w = \beta E_t[\lambda_{t+1}^*]/\lambda_t^*$. These equations both collapse to $q^w = \beta$ in the deterministic steady state.

Admittedly, it is possible to choose a particular steady state amongst the set of possible solutions to the system (11). For example, it is common practice to assume that the symmetric deterministic steady state involves $b = b^* = 0$. Unfortunately, this yields another problem. Namely, the linear dynamic system that describes the behavior of the model's predetermined state variables is not stationary.

To clarify the non-stationarity problem, we apply the numerical linearization method. To do so, we first assign values to all parameters. We follow Backus, Kehoe, and Kydland (1992) and set $\beta = 0.99$, $\gamma = -1$, $\delta = 0.025$, and $\alpha = 0.36$. We set η to ensure that steady state hours worked are $n = 0.3$. We also set ξ to ensure that the ratio of the standard deviations of detrended investment to the standard deviations of detrended output is realistic, where the trend is removed using the Hodrick-Prescott filter. The realistic relative volatility of investment is 3.27.

In addition, we use two di erent parametrizations for the shock process. We do so because Baxter and Crucini (1995) argue that the IM model is very sensitive to the parameters that controls persistence (ρ) and international spillovers (ν). The first parametrization corresponds to the process in Backus, Kehoe, and Kydland (1992). The BKK shock process assumes a small value of ρ and a large value of ν: $\rho = 0.906$ and $\nu = 0.088$. The second parametrization is in the spirit of Baxter and Crucini (1995). The BC shock process assumes a large value of ρ and a small value of ν: $\rho = 0.999$ and $\nu = 0$. The remaining parameters take the values found in Backus, Kehoe, and Kydland (1992): $\sigma = 0.00852$ and $\psi = 0.258\sigma^2$.

We also simplify the system of equations (10) as in Baxter and Crucini (1995). First, we use the home version of equation (10.3) and equation (10.8) to substitute out q_t^w and b_t^*. Second, we use our solution for q_t^w to rewrite the foreign version of (10.3) as $E_t[\lambda_{t+1}]/\lambda_t = E_t\left[\lambda_{t+1}^*\right]/\lambda_t^*$. Third, we sum the home and foreign versions of (10.7) to obtain the goods market clearing condition $c_t + c_t^* + x_t + x_t^* = y_t + y_t^*$, and keep only the home version of (10.7). Finally, we use the home and foreign versions of equation (10.1) to substitute out λ_t and λ_t^*.

Finally, the dynamic system is linearized around the selected deterministic steady state. The system has as many roots outside the unit circle as there are non-predetermined co-state variables. The system thus meets the conditions spelled in Blanchard and Kahn (1980). The solution for the non-predetermined variables and the predetermined state variables are of the form

$$\mathbf{m}_t = \mathbf{A}\mathbf{p}_t + \mathbf{C}\mathbf{z}_t, \tag{12.1}$$

$$\mathbf{p}_{t+1} = \mathbf{B}\mathbf{p}_t + \mathbf{D}\mathbf{z}_t, \tag{12.2}$$

where $\mathbf{m}_t = (\hat{y}_t \quad \hat{y}_t^* \quad \hat{n}_t \quad \hat{n}_t^* \quad \hat{c}_t \quad \hat{c}_t^* \quad \hat{x}_t \quad \hat{x}_t^*)'$ is the vector of non-predetermined variables, $\mathbf{p}_t = (\hat{k}_t \quad \hat{k}_t^* \quad \hat{b}_t)'$ is the vector of predetermined variables, and $\mathbf{z}_t = (\hat{z}_t \quad \hat{z}_t^*)'$ is the vector of productivity shocks. The transformed variables are of the form $\hat{a}_t = \ln(a_t/a) \approx (a_t - a)/a$ where $a_t = (y_t \quad y_t^* \quad n_t \quad n_t^* \quad c_t \quad c_t^* \quad x_t \quad x_t^* \quad k_t \quad k_t^* \quad z_t \quad z_t^*)$, except for $\hat{b}_t = b_t/y$.

The problem is that the roots of the parameter matrix $\mathbf{B}$ show that the system of predetermined variables (12.2) is not stationary. This implies that the system of non-predetermined variables (12.1) is also non-stationary. Specifically, the roots of $\mathbf{B}$ for the where $q^w(s')$ and $b(s'$

are replaced by equations (16) and (17). The CM model system and its companion steady state system have 12 independent equations and must solve for 12 variables. The steady state is thus unique.

For the numerical implementation, we further simplify the system. To do so, we use equation (10.1) to substitute out λ_t and λ_t^*. We also use the parameter values of the baseline model. Note that the CM model has only 2 state variables. The linear solution is as in equations (12), except that $\mathbf{p}_t$

$\theta_0 = 1$ and $\zeta \geq 0$. Also, β_{ct} and β_{nt} are the derivatives of the discount factor $\beta(c_t, n_t)$ with respect to c_t and n_t.

As before, the consumer chooses consumption, employment, and capital and bond holdings to maximize his expected lifetime utility (18) subject to the budget constraint (3) and the accumulation equation (4). The resulting home and foreign bond pricing equations are

$$q_t^w = \beta_t E_t[\lambda_{t+1}]/\lambda_t, \tag{21.1}$$

$$q_t^w = \beta_t^* E_t[\lambda_{t+1}^*]/\lambda_t^*, \tag{21.2}$$

where $\beta_t = \beta(c_t, n_t)$ and $\beta_t^* = \beta(c_t^*, n_t^*)$. In the deterministic steady state, these equations reduce to two independent equations: $q^w = \beta(c, n)$ and $q^w = \beta(c^*, n^*)$. The deterministic steady state is thus unique.

We implement our numerical linearization method as in the IM model, with one exception. This version of the model replaces the parameter β with the function $\beta(c, n)$, which contains the parameter ζ. We set ζ to ensure that the steady state value of $\beta(c, n) =$ 0.99 as in the IM model. For this version, the roots of the parameter matrix $\mathbf{B}$ for the BKK process are (0.884, 0.959, 0.996). The roots for the BC process are (0.929, 0.962, 0.996). The linear system is thus stationary.

3.2 The Endogenous Discount Factor without Internalization (DFwI) Model

The DFwI model also assumes that the consumer's subjective discount factor is endogenous. The discount factor depends on aggregate consumption and aggregate employment, and the consumer does not internalize the e ects of his choices on the discount factor. A similar assumption is used in Schmitt-Grohé and Uribe (2003).

The consumer's expected lifetime utility is as in (18), but the discount factor is given by

$$\theta_{t+1} = \beta(\tilde{c}_t, \tilde{n}_t)\theta_t, \tag{22}$$

where $\tilde{c}_t$ and $\tilde{n}_t$ are the average per capita consumption and employment in the country. As before, the consumer chooses consumption, employment, and capital and bond holdings

As required, the deterministic steady state of these equations imply two independent equations: $q = \beta$ and $q^* = \beta$, while the steady state of equation (24) yields $(R^* - R)b = \varphi RR^*(b^2/y + b^{*2}/y^*)$. The steady state is thus unique.

We implement our numerical linearization method as in the IM model. We set the responsiveness of the real interest rate di erential to changes in the net foreign asset position to the value found in Lane and Milesi-Ferreti (2002). In the steady state, the responsiveness is φ/β^2 since $R = R^* = 1/\beta$. Thus, we set $\varphi = \beta^2 \quad 0.01$. The resulting roots of the parameter matrix $\mathbf{B}$ with the BKK process are (0.600, 0.965, 0.967) and with the BC process are (0.577, 0.965, 0.967). The linear system is thus stationary.

3.4 The Quadratic Portfolio Costs (QPC) Model

The QPC model assumes quadratic portfolio costs, as in Heathcote and Perri (2002). These costs are motivated by small costs to buying the bond. In this case, the consumer's budget constraint is

$$c_t + x_t + q_t^w b_{t+1} + \frac{\pi}{2} b_{t+1}^2 = w_t n_t + r_t^k k_t + b_t, \tag{26}$$

where $\pi \geq 0$.

The consumer chooses consumption, employment, and capital and bond holdings to maximize expected lifetime utility (1) subject to the budget constraint (26) and the accumulation equation (4). The home and foreign bond pricing equations are

$$q_t^w = \beta E_t[\lambda_{t+1}]/\lambda_t - \pi b_{t+1}, \tag{27.1}$$

$$q_t^w = \beta E_t[\lambda_{t+1}^*]/\lambda_t^* - \pi b_{t+1}^*. \tag{27.2}$$

The deterministic steady state of equations (27) yields two independent equations: $q^w = \beta - \pi b$ and $q^w = \beta - \pi b^*$. The steady state is thus unique.

We implement our numerical linearization method as in the IM model. We set $\pi = \beta^2 \quad 0.01$ to ensure that the QPC model is comparable to the DER model. The resulting roots of the parameter matrix $\mathbf{B}$ with the BKK process are (0.578, 0.965, 0.967) and with the BC process are (0.233, 0.965, 0.967). The linear system is thus stationary.

3.5 The Direct Preferences for Wealth (DPW) Model

The DPW model assumes that consumers care about their relative wealth, as in Gong and Zou (2002) and Fisher and Hof (2005). The consumer's expected lifetime utility is

$$E_0 \sum_{t=0}^{\infty} \beta^t \mathrm{v}$$

4. Numerical Results

Baxter and Crucini (1995) argue that the IM model generates a differential wealth effect that explains the lack of international risk sharing found in the data and that solves the quantity anomaly. This effect, however, is sensitive to the parametrization of the shock process. It occurs only when shocks are highly persistent and do not spill over international boundaries.

In this section, we verify whether the differential wealth effect and the ability to solve the quantity anomaly survive in stationary incomplete markets IRBC models. For this, we compare the business cycle moments and impulse responses of the different models driven by both BKK and BC parametrizations of the shock process.

4.1 Business Cycle Moments and Impulse Responses

We first compare a number of business cycle moments and impulse responses generated by the various models for the two different parametrizations of the shock process. Table 1 and Figure 1 present the moments and responses for the models driven by the BKK shock the DF and DFwI models, the exercise requires that the discount factor be restated as $\beta(c_t, n_t) = \bar{\beta}[1 + c_t^{\eta}(1 - n_t)^{1-\eta}]^{-\zeta}$

and DPW models are far from those of the data. The DF and DFwI models generate too much volatility for consumption and for the net export to output ratio. In addition, they grossly understate the extent to which employment is procyclical. The DPW model generates too much volatility for the net export to output ratio. Finally, only the DER and QPC models produce a positive cross-country correlation of consumption. In fact, for these two models, the ratio of the cross-country correlations of consumption and output is roughly 77 percent, as in the data. Third, the responses of the IM and stationary models also differ. The responses of output and consumption appear similar across the different models, but the responses of investment, employment, and the net exports to output ratio differ considerably. Notably, the DF and DFwI models predict a reduction of employment following the positive productivity shock, and the DPW model generates large fluctuations for the net export to output ratio.

Overall, the ability to solve the quantity anomaly is shared by all incomplete financial of consumption and employment into wealth and price effects (wage and interest rate). The decomposition is computed as follows. Consider the responses of consumers to a positive innovation to home productivity, as shown in Figure 1 and 2. First, for given prices, consumers alter their consumption and employment choices because the higher productivity changes their wealth. The wealth effect is measured as the constant responses of consumption and employment that produce a change in the lifetime utility identical to that produced by the home productivity shock, holding prices constant. Second, for given wealth, consumers also alter their choices because the higher productivity changes both the wage rate and the interest rate that they face. The wage rate effect is measured and a much smaller positive wealth effect abroad. This largely raises home consumption and reduces home employment. It also slightly raises foreign consumption and reduces foreign employment.

As in Baxter and Crucini (1995), the differences in the wealth effects explains why incomplete markets resolve the quantity anomaly. Under complete markets, the rise in

4.3 Extensions

The Hicksian decompositions document that the difference between the baseline IM model and the DER model lies in the price effects. This suggests that the general equilibrium

This occurs because of the price effects, and especially of the interest rate effect.

5. Conclusion

Several authors argue that the baseline IRBC model with incomplete international financial markets provides a solution to the *quantity anomaly*. For this, productivity shocks must be highly persistent and must not spill over international boundaries.

Unfortunately, the above conclusion is suspect because it stems from an analysis of the near steady state dynamics using a linearized system of equations. The baseline IRBC model with incomplete financial markets does not possess a unique deterministic steady state and, as a result, its linear system of difference equations is not stationary.

We show that the ability to solve the quantity anomaly is robust to modifications of the model that ensure the existence of a unique steady state and a stationarity system of linear difference equations. We find, however, that the modifications affect the quantitative predictions regarding key macroeconomic variables, especially when the model solves the quantity anomaly.

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A. Technical Appendix

In this appendix, we present the system of equations that characterizes the equilibrium for each stationary incomplete markets model.

A.1 The Endogenous Discount Factor (DF) Model

The system of equations that characterizes the equilibrium of the DF model includes home and foreign variants of

$$\lambda_t = u_{ct} - \beta_{ct} a_t, \qquad (A1.1)$$

$$u_{nt} = -\lambda_t (1 - \alpha$$

as well as the asset market clearing condition

$$b_t + b_t^* = 0, \qquad (A2.8)$$

where $\tilde{c}_t = c_t$ and $\tilde{n}_t = n_t$. The system (A2) and its companion deterministic steady state system both have 15 independent equations and must solve for 15 variables. The solution to the steady state is unique.

A.3 The Debt Elastic Interest Rate (DER) Model

The system of equations that characterizes the equilibrium of the DER model includes home and foreign variants of

$$\lambda_t = u_{ct}, \qquad (A3.1)$$

$$u_{nt} = -$$

as well as the asset market clearing condition

$$b_t + b_t^* = 0. \tag{A4.8}$$

The system (A4) and its companion deterministic steady state both have 15 independent equations and must solve 15 variables. Thus the deterministic steady state is unique.

A.5 The Direct Preferences for Wealth (DPW) Model

The system of equations that characterizes the equilibrium of the DPW model includes home and foreign variants of

$$\lambda_t = \mathrm{v}_{ct}, \tag{A5.1}$$

$$\mathrm{v}_{nt} = -\lambda_t(1-\alpha)y_t/n_t, \tag{A5.2}$$

$$q_t^w = \beta E_t\left[\lambda_{t+1} + \mathrm{v}_{vt+1}/v_{t+1}^w\right]/\lambda_t, \tag{A5.3}$$

$$\frac{\lambda_t}{\phi_{\delta t}} = \beta E_t\left[\lambda_{t+1}\alpha\frac{y_{t+1}}{k_{t+1}} + \frac{\lambda_{t+1}}{\phi_{\delta t+1}}\left(\phi_{t+1} - \phi_{\delta t+1}\frac{x_{t+1}}{k_{t+1}} + (1-\delta)\right) + \frac{\mathrm{v}_{vt+1}}{v_{t+1}^w}\right], \tag{A5.4}$$

$$y_t = z_t k_t^\alpha n_t^{1-\alpha}, \tag{A5.5}$$

$$k_{t+1} = \phi_t k_t + (1-\delta)k_t, \tag{A5.6}$$

$$y_t = c_t + x_t + q_t^w b_{t+1} - b_t, \tag{A5.7}$$

$$v_t = (k_t$$

Table 1. Business Cycle Moments with BKK Shock Process

	Data	CM	IM	DF	DFwI	DER	QPC	DPW
Standard deviations relative to output:								
Consumption	0.75	0.45	0.46	0.42	0.48	0.49	0.49	0.43
Investment	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27
Employment	0.61	0.48	0.46	0.50	0.45	0.41	0.41	0.44
Net exports/output	0.27	0.31	0.32	0.32	0.33	0.29	0.29	0.76
Correlations with output:								
Past output	0.86	0.69	0.69	0.69	0.69	0.69	0.69	0.69
Consumption	0.82	0.82	0.85	0.79	0.85	0.92	0.92	0.92
Investment	0.94	0.90	0.89	0.89	0.88	0.90	0.90	0.83
Employment	0.88	0.93	0.93	0.93	0.92	0.94	0.94	0.96
Net exports/output	-0.37	-0.09	-0.11	-0.01	-0.12	-0.30	-0.30	-0.44
Cross-country correlations:								
Output	0.66	-0.03	-0.01	-0.04	-0.03	0.07	0.07	0.07
Consumption	0.51	0.91	0.87	0.94	0.87	0.72	0.72	0.73

Note: Entries under standard deviations relative to output are the ratio of the standard deviation of a variable to that of the logarithm of output. Entries under correlations with output are the contemporaneous correlation between a variable and the logarithm of output. Entries under cross-country correlations are the contemporaneous correlation between home and foreign variables. The variables are the logarithm of output, the logarithm of consumption, the logarithm of investment, the logarithm of employment, and the ratio of net exports and output. All variables are detrended with the Hodrick-Prescott filter. The Data column is taken from Backus, Kehoe, and Kydland (1995), and it refers to U.S. data and U.S. and Europe data for the period 1970:I to 1990:II. Also, CM stands for the complete markets model, IM for the baseline incomplete markets model, DF for the endogenous discount factor model, DFwI for the endogenous discount factor without internalization model, DER for the debt elastic interest rate model, QPC for the quadratic portfolio costs model, and DWP for the direct preferences for wealth model.

Table 2. Business Cycle Moments with BC Shock Process

	Data	CM	IM	DF	DFwI	DER	QPC	DPW
Standard deviations relative to output:								
Consumption	0.75	0.52	0.96	1.11	1.11	0.70	0.70	0.78
Investment	3.27	3.27	3.27	3.27	3.27	3.27	3.27	3.27
Employment	0.61	0.47	0.25	0.43	0.37	0.22	0.22	0.23
Net exports/output	0.27	0.24	0.82	1.03	1.00	0.57	0.57	1.06
Correlations with output:								
Past output	0.86	0.72	0.72	0.72	0.72	0.72	0.72	0.72
Consumption	0.82	0.78	0.93	0.84	0.88	0.99	0.99	0.96
Investment	0.94	0.90	0.81	0.77	0.76	0.78	0.78	0.79
Employment	0.88	0.88	0.30	0.12	0.04	0.98	0.98	0.77
Net exports/output	-0.37	-0.24	-0.42	-0.33	-0.36	-0.30	-0.30	-0.46
Cross-country correlations:								
Output	0.66	-0.15	0.48	0.65	0.60	0.22	0.22	0.39
Consumption	0.51	0.92	-0.13	-0.32	-0.24	0.17	0.17	-0.06

Note: Entries under standard deviations relative to output are the ratio of the standard deviation of a variable to that of the logarithm of output. Entries under correlations with output are the contemporaneous correlation between a variable and the logarithm of output. Entries under cross-country correlations are the contemporaneous correlation between home and foreign variables. The variables are the logarithm of output, the logarithm of consumption, the logarithm of investment, the logarithm of employment, and the ratio of net exports and output. All variables are detrended with the Hodrick-Prescott filter. The Data column is taken from Backus, Kehoe, and Kydland (1995), and it refers to U.S. data and U.S. and Europe data for the period 1970:I to 1990:II. Also, CM stands for the complete markets model, IM for the baseline incomplete markets model, DF for the endogenous discount factor model, DFwI for the endogenous discount factor without internalization model, DER for the debt elastic interest rate model, QPC for the quadratic portfolio costs model, and DWP for the direct preferences for wealth model.

Table 3. Moments in Alternative Models with BC shock process

		IM		DER	
	Data	No N	No K	No N	No K
Standard deviations relative to output:					
Consumption	0.75	0.94	0.95	0.72	1.00
Investment	3.27	3.27	0.00	3.27	0.00
Employment	0.61	0.00	0.06	0.00	0.01
Net exports/output	0.27	0.72	0.08	0.57	0.01
Correlations with output:					
Past output	0.86	0.72	0.70	0.72	0.70
Consumption	0.82	0.97	1.00	0.99	1.00
Investment	0.94	0.86	0.00	0.78	0.00
Employment	0.88	0.00	0.63	0.00	0.57
Net exports/output	-0.37	-0.55	0.63	-0.33	0.57
Cross-country correlations:					
Output	0.66	0.22	0.20	0.21	0.25
Consumption	0.51	-0.23	0.32	0.16	0.26

Note: Entries under standard deviations relative to output are the ratio of the standard deviation of a variable to that of the logarithm of output. Entries under correlations with output are the contemporaneous correlation between a variable and the logarithm of output. Entries under cross-country correlations are the contemporaneous correlation between home and foreign variables. The variables are the logarithm of output, the logarithm of consumption, the logarithm of investment, the logarithm of employment, and the ratio of net exports and output. All variables are detrended with the Hodrick-Prescott filter. The Data column is taken from Backus, Kehoe, and Kydland (1995), and it refers to U.S. data and U.S. and Europe data for the period 1970:I to 1990:II. Also, IM stands for the baseline incomplete markets model and DER for the debt elastic interest rate model. Under both IM and DER, No N stands for inelastic labor and No K for inelastic capital.

Figure 1. Dynamic Responses with BKK Shock Process

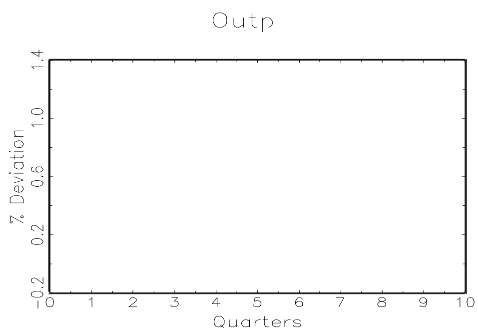

Figure 2. Dynamic Responses with BC Shock Process

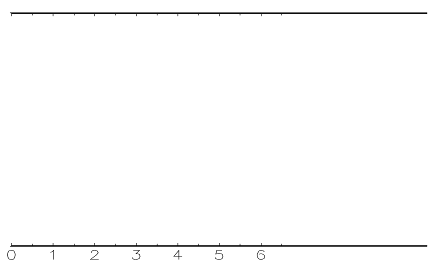

Figure 3. Decomposition of Consumption and Employment Responses

CM with BC Shock Process

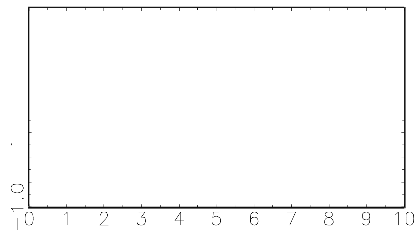

Figure 4. Decomposition of Consumption and Employment Responses

IM with BC Shock Process

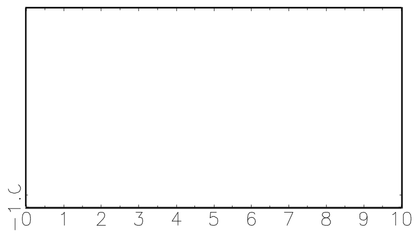

Figure 5. Decomposition of Consumption and Employment Responses

DER with BC Shock Process

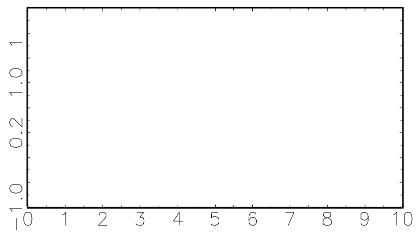

Figure 6. Decomposition of Consumption Responses

IM and DER with No N and BC Shock Process

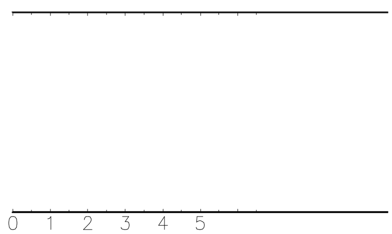

Figure 7. Decomposition of Consumption Responses

IM and DER with No K and BC Shock Process