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Relative Economic Efficiency  
and the Provision of Rationed Goods

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## Abstract:

Public lands and rivers currently support many recreational activities for which demand seriously exceeds supply. Almost all of these recreational opportunities, such as hunting and rafting permits, are allocated either through lottery, queue, or some combination of the two. Clearly, the current allocation is economically inefficient since low- and high-value users are equally likely to receive permits. Political opposition prevents the resource manager from exclusive use of market allocations. We present a simple relative efficiency measure for evaluating the economic efficiency of alternative allocations. We also evaluate alternative allocations in which some of the available permits are distributed via auction, the remaining via lottery. The auction/lottery

## 1 Introduction

Public lands and rivers in the United States currently support many recreational activities for which demand seriously exceeds the supply. Prominent examples include rafting along the Colorado River in the Grand Canyon, which now has a 14 year queue for individual rafting permits, and hunting, where in some states the number of permit applicants is 165 times the number of permits issued. Almost all of these recreational opportunities are allocated to the public either through lottery, queue, or some combination of the two. In the case of big game hunting permits, some states offer a very small number of permits, as few as a single permit, for sale through an auction. The available auction data show that many of these quantity rationed resources are *very highly* valued. For example, in a 1998 auction for a Calgary bighorn sheep permit, the winning bidder paid \$405,000 US.

Economic efficiency requires that these resources flow to their most highly valued use. A properly functioning market could easily obtain an economically efficient allocation of these resources. However many citizens, even nonusers, oppose market allocation of these publicly provided goods. Typically, opponents of market allocation cite concerns over equity as their primary reason for opposition.

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<sup>1</sup>There exists opposition to market allocation even within the field of economics. Nickerson (1990) notes “markets that allocate by willingness to pay are not usable or even desirable distribution mechanisms for allocating publicly managed goods.”

over equity may very well be justified. More importantly, it is easy to comprehend why so many users oppose market allocation. Under the current allocation systems, all applicants are participating in lotteries or queues for which the expected returns are positive, otherwise the applicants would not participate. One can see by the sheer excess demand numbers alone that moving to market allocation could potentially result in welfare losses, at least in expectation, for hundreds of thousands of recreationists and hunters. Thus it is perfectly rational for users to oppose market allocation and hence for us to see the political equilibrium that has resulted from greatly increased demand for these resources over the years. Economists should not expect resource managers to warmly embrace the notion of market allocation simply because it increases economic efficiency. A unilateral move to a market allocation by an individual resource manager predictably would lead to a user rebellion, a move tantamount to professional suicide by the resource manager.

While the constraints placed on allocation options by the public are real and pressing, it is in both the resources manager's and the public's interest to consider the opportunity cost of the current allocation systems. Nickerson (1990) advocates more careful analyses of outdoor regulation in order to "reduce the costs of the regulatory process both monetarily and politically." He suggests that these analyses would afford resource managers better information on the effectiveness of various policies on the effectiveness of various professional suicides

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for rafting on the Colorado River in the Grand Canyon. According to the Grand Canyon Private Boaters Association, “Because of its reputation as the premier whitewater experience, no other river is more in demand...”<sup>3</sup> Depending upon user type, potential users enter one of two user pools, commercial or non-commercial. Permits are then allocated via queue within each user pool. Non-commercial or private permit applicants pay a \$100 application fee to have their name placed on the wait list. Each year the applicant must submit a form of continuing interest in order to maintain his place in the queue. In addition to the application fees and the travel cost of the trip, those obtaining permits must pay a \$100 permit fee per trip participant. Non-commercial applicants who enter the queue in 2000 can expect to wait approximately 14 years before obtaining a permit.<sup>4</sup> In addition to the queue there is an auxiliary system for cancellations. Each week hopeful applicants can call the agency to find out if there has been a recent cancellation. While the cancellation system favors individuals higher in the queue, there is not a secondary list for cancellations. Obtaining a cancellation permit is more a matter of luck and persistence in calling the cancellation line.

Approximately 75% of all user days are allocated to commercial outfitters, leaving only 25% for private individuals. Private individuals can also gain access to the river by signing up with a commercial outfitter. Commercial trips must be guided by a certified guide and so a private paddler is not on her own. For this reason, the serious paddler does not view private and commercial trips as perfect substitutes. For the Grand Canyon, there may be substantial efficiency and revenue gains to selling some of the commercial user days to private users.

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<sup>3</sup>This information was obtained from <http://www.gcpba.org>.

<sup>4</sup>Commercial applicants typically experience a maximum wait between one and two years.


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<sup>5</sup>This information was obtained from <http://www.dnr.state.co.us/wildlife/hunt>.

<sup>6</sup>Buschena, Anderson et al. (2001) offer a more detailed discussion of the allocation of elk hunting permits in Colorado and introduce a method of inferring permit values under this system.

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<sup>7</sup>The Foundation for North American Wild Sheep (



through lottery, queue, or lottery/queue combination. In order to analyze the economic efficiency of the allocation, we require information about the economic valuations of the larger group of resource users, not just the auction winners. The Foundation data only contains the winning bids.

The State of Maine auctions moose hunting permits annually.<sup>10</sup> In recent years Maine has offered five permits through its annual auction. Unlike the Foundation auction, the Maine auction uses a discriminative auction in which bidders submit their bids via mail. After the auction takes place, Maine holds a random lottery for the remaining moose hunting permits, typically about 2000 per year. The Maine auction procedure has the advantage over the Foundation auction in that it yields bid data for all those participating in the auction. In the next section, we propose a way of using this data to gauge the economic efficiency of the current or a proposed allocation.

### **3 Relative Efficiency Measure and Total Value Discussion**

Now let us consider the problem from the perspective of the resource manager. Given that the good is not currently provided in the market and there exists excess demand at the current permit price, permits are effectively quantity rationed. The resource manager must decide how to allocate the rationed good among those who value the permits. Specifically, he must determine how to allocate  $\rho$  permits/goods, which we assume to be identical, among  $n$

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<sup>10</sup>Maine currently auctions its own permits. In 1996 and 1997 the Foundation for North American Wild Sheep auctioned a single permit for Maine in each respective year.

consumers where  $n \gg \rho$ .<sup>11</sup> As discussed above, he must consider both issues of efficiency and equity/political feasibility when considering different allocations. As economists, we restrict our attention to the question of economic efficiency. We develop an efficiency measure which may be used by the resource manager, along with other considerations of equity/political feasibility, to determine the appropriate allocation.

In order to develop this efficiency measure, we first establish upper and lower bounds on the total value ( $TV$ ) of any given allocation. The total value of an allocation is the sum of the monetary valuations of those individuals who receive permits under that particular arrangement. The monetary valuation is simply the maximum willingness to pay, also referred to in the economics literature as compensating surplus, for a permit. Note that total value can be calculated with precision only when we know each individual's valuation for the good,  $v_i$  where  $i = 1, \dots, n$ . Without loss of generality, we refer to individuals by the descending rank of their valuations,  $v_1 > v_2 > \dots > v_n$ . We use this notation in discussing the total value associated with various allocations.

Total value is maximized, *i.e.* economic efficiency is obtained, when the resource manager auctions off all  $\rho$  permits. This pure auction has the benefit of encouraging individuals to reveal information about their valuations.<sup>12</sup> Under fairly general conditions, perfect revelation in an auction is incentive compatible. Vickrey (1961) explores the Nash equilibrium bidding

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<sup>11</sup>While an interesting issue, we do not address the optimal choice of  $\rho$ . We assume that the number of permits to be allocated,  $\rho$ , is predetermined and fixed. Sandrey, Buccola et al. (1983) similarly assume a predetermined supply in their analysis of elk hunting permits..

<sup>12</sup> We use the term "pure auction" to indicate an auction of all available permits.

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<sup>13</sup> A multiple unit auction in which the price paid by all winners is equal to the first bid rejected is commonly referred to as a competitive uniform price auction.

via market-based methods.

Because the resource manager is constrained in the number of permits he may auction off, he is unable to achieve economic efficiency as obtained with a pure auction. We can calculate a *feasible* upper bound on the total value of any allocation.<sup>14</sup> Let the *feasible pure auction* be an auction in which all  $I$  permits are allocated via auction. The total value of the feasible pure auction (FPA) allocation is given by:

$$(1) \quad TV_{FPA} = \sum_{i=1}^I v_i$$

The feasible pure auction case serves as a meaningful benchmark for evaluating other potential allocations because it serves as an upper bound on the total value for any other allocation.

We can construct a measure of the relative efficiency of a given allocation of the  $I$  permits,  $m$ , by examining the total value of the feasible pure auction and the total value of the alternative allocation,  $TV_m$ , in ratio form. The relative efficiency of allocation  $m$  is given by the following expression:

$$(2) \quad e_m = \frac{TV_m}{TV_{FPA}} = \frac{TV_m}{\sum_{i=1}^I v_i}$$

Our measure falls in the unit interval,  $0 < e_m \leq 1$ , and measures the percentage of maximum

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<sup>14</sup> Note that the true upper bound is the total value when the resource manager auctions off all  $I$  permits. Because the manager is restricted, in practice, from choosing this allocation, the true pure auction has limited value as a benchmark. Also, in order to calculate the total value of this allocation, we must have valuation information on at least the  $I$  highest valued users. As will be illustrated in the next section, this information requirement is rarely satisfied.

surplus obtained by allocation  $m$ , hence the term relative economic efficiency. The relative efficiency measure equals one when  $m$  is the feasible pure auction, but will be less than one for

$$(3) \quad E [ TV_L ] = \frac{1}{n} \sum_{i=1}^n v_i = \mu_n$$

$$(4) \quad \hat{TV}_L = \frac{1}{n'} \sum_{i=1}^{n'} v_i = \mu_{n'}$$

Provided  $\mu_{n'} > \mu_n$ ,  $T\hat{V}_L$  will overestimate the true total value of the lottery. Under the realistic assumption that not all individuals have identical valuations for the permits,  $TV_{FPA} > T\hat{V}_L$ .

Using the sample bid information, we form an estimate of the expected relative efficiency of the lottery format given by:

$$(5) \quad \hat{e}_L = \frac{\frac{1}{n'} \sum_{i=1}^{n'} v_i}{\sum_{i=1}^{n'} v_i} = \frac{1}{TV_{FPA}} \mu_{n'}$$

The expectation of the relative efficiency of the lottery is random only in the numerator; the denominator is always given by the sum of the  $n'$  largest values. Similarly, the estimate of expected relative efficiency of the feasible pure auction is always 1. If we consider allocation by a combination of auction and lottery, then the expected relative efficiency of any combination allocation is bounded above by one and below by  $\hat{e}_L$ .

### 3.1 The Combination Auction/Lottery as an Information Source

From the perspective of the resource manager, both the feasible pure auction and pure lottery have advantages and disadvantages. The feasible pure auction has the advantage of maximizing total value but may have political costs. The feasible pure auction also acts as an information source of values. The pure lottery, on the other hand, is desirable in terms of equity but can potentially yield a very economically inefficient outcome if the distribution of individual values is right-skewed. In addition, the pure lottery fails to reveal information about individual valuations. In order to capture some of the respective benefits of both pure allocation methods, we consider an alternative scheme that combines the two mechanisms. The auction allows some

of the permits to go to the highest valued uses while still providing a lottery for some and

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<sup>15</sup>We believe this because even with a large valuation, the expected return from the lottery is fairly small when  $n$  is large.

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<sup>16</sup>When  $n$  is large and  $j$  is small, the mean of the entire population,  $\mu$  should be fairly close to .



not yet know the identities of the lottery winners.<sup>17</sup> If everyone entered the auction in the first stage, there would be individual valuations for all users and so the resource manager could calculate the expected total value for the combination as well as the expected relative efficiency as a function of the number of permits auctioned.<sup>18</sup>

$$(7) \quad E [ e(j) ] = \frac{(n' - j)\mu_{n-j} + \sum_{i=1}^j v_i}{\sum_{i=1}^{n'} v_i}$$

As discussed earlier, the resource manager rarely has value information for all  $n$  individuals. As in the application presented later, resource managers often have access to bid values for only a subset of lottery entrants. Provided this is the case, he can calculate an estimate of  $E[e(j)]$  and use this estimate to compare different allocations. Suppose the resource manager has bids for only the top  $n' < n$  lottery entrants. In other words, only a portion of the lottery entrants enter the auction. In this case,  $\mu_n$ , the population mean is unknown. The resource manager can, however, use the information contained in the sample of  $n'$  bidders to examine the efficiency of different allocations. Let  $\mu_{n'}$  represent the mean bid of the auction entrants and  $\mu_{n'-j}$  be the auction sample mean when the top  $j$  bidders are removed. Using the auction sample bid data, we can estimate the total value of the allocation as follows:

$$(8) \quad \hat{TV}_{A/L} = \frac{n' - j}{n' - j} \sum_{i=j+1}^{n'} v_i + \sum_{i=1}^j v_i = (n' - j) \mu_{n'-j} + \sum_{i=1}^j v_i$$

<sup>17</sup> The total value of the auction/lottery combination can also be thought of as an expected total value where the expectation is taken after the auction stage.

<sup>18</sup>If everyone submitted a bid, the resource manager would know the exact total value since there would be information on all users. In considering alternative allocations, the expected value would be a more useful predictor since we know that it minimizes predicted mean square error.

The estimate of relative efficiency is then given by:

$$(9) \quad \hat{e}(j) = \frac{(\cdot - j) \mu_{n'-j} + \sum_{i=1}^j v_i}{\sum_{i=1}^j v_i}$$

Without bid information on all  $n$  lottery entrants, we are unable to calculate the actual expected efficiency measure but we can obtain an estimate by using the bid data contained in the auction sample. Fortunately, we can determine the direction and magnitude of the bias for each of our estimates.<sup>19</sup> Our estimate slightly overestimates the total value and relative efficiency

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<sup>19</sup> See the appendix for the derivations.

<sup>20</sup> These results require that  $\mu_{n'-j} \approx \mu_{n'-(j-1)}$ .

### 3.3 Case 2: Lottery/Auction

Consider a variation of the two-step allocation method where the lottery takes place in the first stage and the auction in the second. Since the denominator of the relative efficiency measure is constant, we will limit discussion to the actual total value of this mechanism. Note that this differs from the discussion of the auction/lottery mechanism where we developed an expected total value measure. Continue to assume that bidder  $i$  submits a bid equal to his valuation. In order to derive an expression for the total value of the lottery/auction combination, we first examine the total value added in each stage individually. In the first stage, the resource manager allocates  $k = \ell - j$  permits via lottery. Letting  $W$  represent the set of  $k$  lottery

$$(10) \quad TV_{LA}^{SI} = \sum_{i \in W} v_i$$

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<sup>21</sup>Note that this is the actual total value of the first stage, not the expected total value as before. The (expected) total value of the first stage is given by  $E [TV_{LA}^{SI}] = (\ell - j) \mu_n$ .

$$(11) \quad TV_{LA}^{S2} = \sum_{i=1}^j v_i'$$

Summing the total values of both stages, we get an expression for the total value of the lottery/auction combination.

$$(12) \quad TV_{LA} = \sum_{i \in W} v_i + \sum_{i=1}^j v_i'$$

This measure has limited practical use for the resource manager wishing to evaluate alternative allocations since it requires that he know which bidders will win the lottery and thus be eliminated from the auction.<sup>22</sup> We can, however, approximate the total value of the lottery/auction with the estimate of expected total value of the auction/lottery developed earlier. In large samples, when  $j$  is small, the total values of the auction/lottery and lottery/auction are approximately equal. The two mechanisms may differ considerably along the lines of equity/political feasibility, a consideration that the resource manager can ponder when choosing the most appropriate allocation.<sup>23</sup> If he chooses either combination mechanism, he must also choose  $j$ , the number of permits to be allocated via auction. In the next section, we use sample

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<sup>22</sup>One can derive an expression for the expected total surplus from the lottery/auction combination. This expression is complicated because there are  $n$  choose  $j$  distinct outcomes from the lottery, occurring with equal probability. With heterogeneous values, each outcome has its own realized total value. For this reason we do not present expected total surplus or expected efficiency.

<sup>23</sup>When  $n$  is large relative to  $j$ , most individuals should prefer the auction/lottery since most will not gain from the auction, regardless if held before or after the lottery. Allowing the auction to come first increases the probability of being chosen in the lottery from  $1/n$  to  $1/(n-j)$  and so most should prefer the auction/lottery over the lottery/auction. Only individuals  $i = 1, 2, \dots, j$  can be positively affected by the auction in the lottery/auction combination. Individual  $i$  could win in the auction only if lottery winners all had higher valuations than  $v_i$ .




Second, a significant portion of lottery entrants failed to enter the auction. We suspect that individuals who entered both the auction and lottery have high values relative to those who entered only the lottery. Since the expected total value from the lottery portion of the combination is derived from the auction data, we will overestimate the total value from the lottery stage which manifests itself in an overestimate of relative efficiency.

Assume that the resource manager has decided to allocate a maximum of 14 permits, or 1% of the total number of permits, via auction. This suggests that, for our example,  $j = 14$ . Having chosen the maximum number of permits to be auctioned off, the resource manager's interest lies in the total value of various allocations. He can also examine the marginal efficiency of various choices of  $j$ , the number of permits auctioned, in order to make a more informed allocation decision.

Before examining the relative efficiency of the auction/lottery combination, however, we develop bounds on the efficiency measure. The total value of the feasible pure auction is equal to the sum of the top 14 bids,  $TV_{FPA} = \$79,555.50$ . The expected total value of a pure lottery is given as follows. Our estimate of  $\mu_n$  is the mean of the auction sample,  $\mu_{n'}$ , which equals \$1956.62. Thus our estimate of the expected total surplus from the data is  $T\hat{V}_L = 14\mu_{n'} = \$27,392.68$ . Therefore, we expect the relative efficiency measure of an alternative

$$(13) \quad \hat{e}_L \approx 0.3443 \leq \hat{e}_m \leq 1$$

choices of  $j$ . We can think about the pure lottery and feasible pure auction as special cases of the combination mechanism with  $j = 0$  and  $j = 14$  respectively. We can calculate the efficiency






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auction winners is equal to the first bid rejected. The final column of Table 4 displays the marginal revenue generated for different choices of  $j$ . While revenue generation is unlikely to be his main concern, the resource manager will certainly value access to revenue information when choosing an allocation.<sup>25</sup>

The resource manager, when determining how to regulate access to the resource, must consider both economic efficiency and equity/political feasibility associated with potential allocations. His job is complicated further in the presence of excess demand for the activity. The method presented here, when combined with information about equity and revenue generation, aides the resource manager in making a more informed allocation decision. We provide a method for determining the relative efficiency gain (loss) associated with various allocations.

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<sup>25</sup> Sandrey, Buccola et al. (1983) suggest that a decreased emphasis on low cost allocation policies may allow state fish and wildlife departments to become more self-supporting, and therefore less reliant on state and federal support.

$$(14) \quad TV_{A/L} = \frac{l-j}{n-j} \sum_{i=j+1}^n v_i + \sum_{i=1}^j v_i \equiv (l-j) \mu_{n-j} + \sum_{i=1}^j v_i$$

$$T\hat{V}_{A/L} = \frac{l-j}{n'-j} \sum_{i=j+1}^{n'} v_i + \sum_{i=1}^j v_i$$

$$e(j) - \hat{e}(j) = \frac{(l - j)(\mu_{n-j})}{\dots}$$

7. Figures

Figure 1 Relative Efficiency of Auction/Lottery Combination

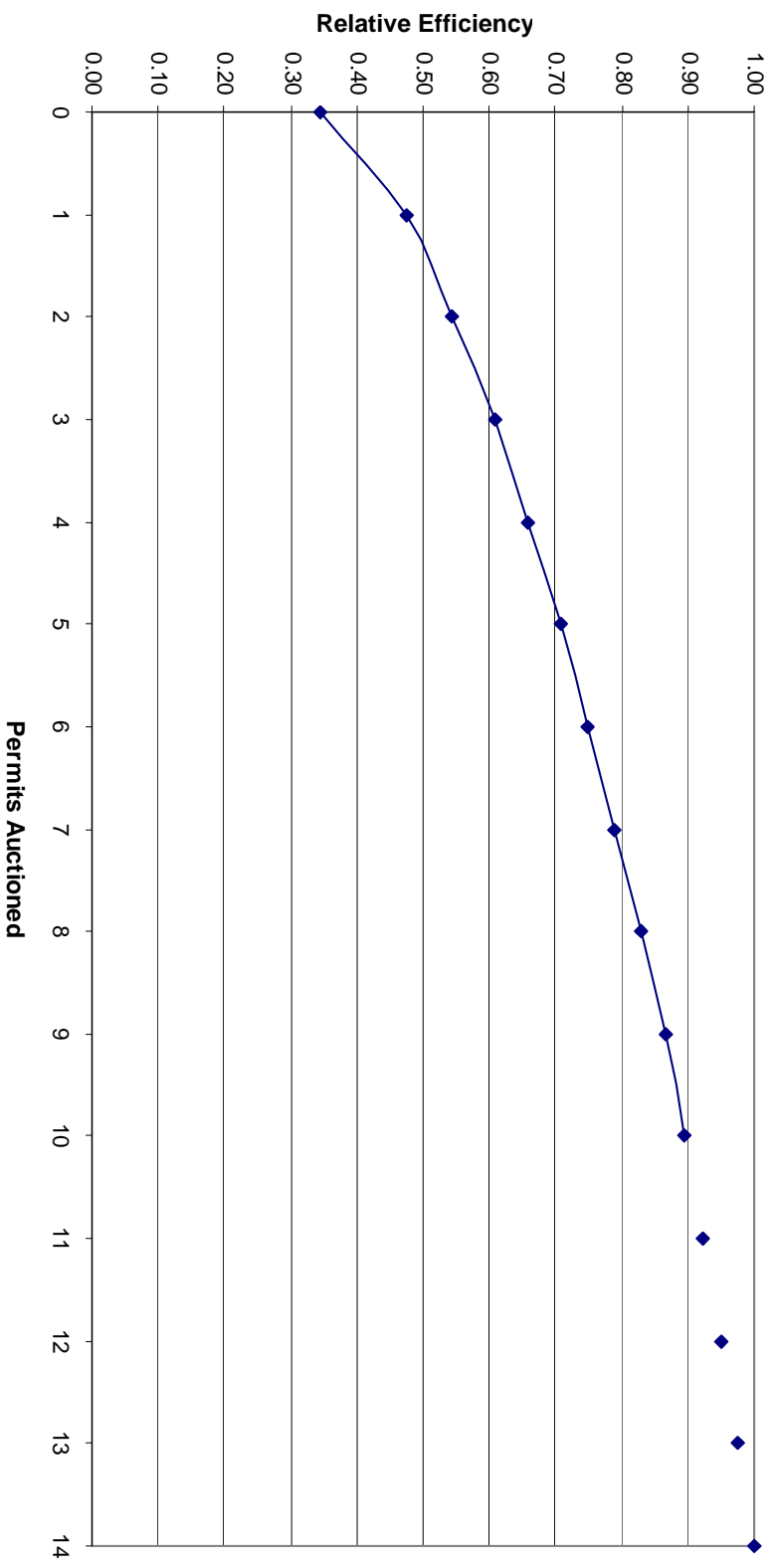
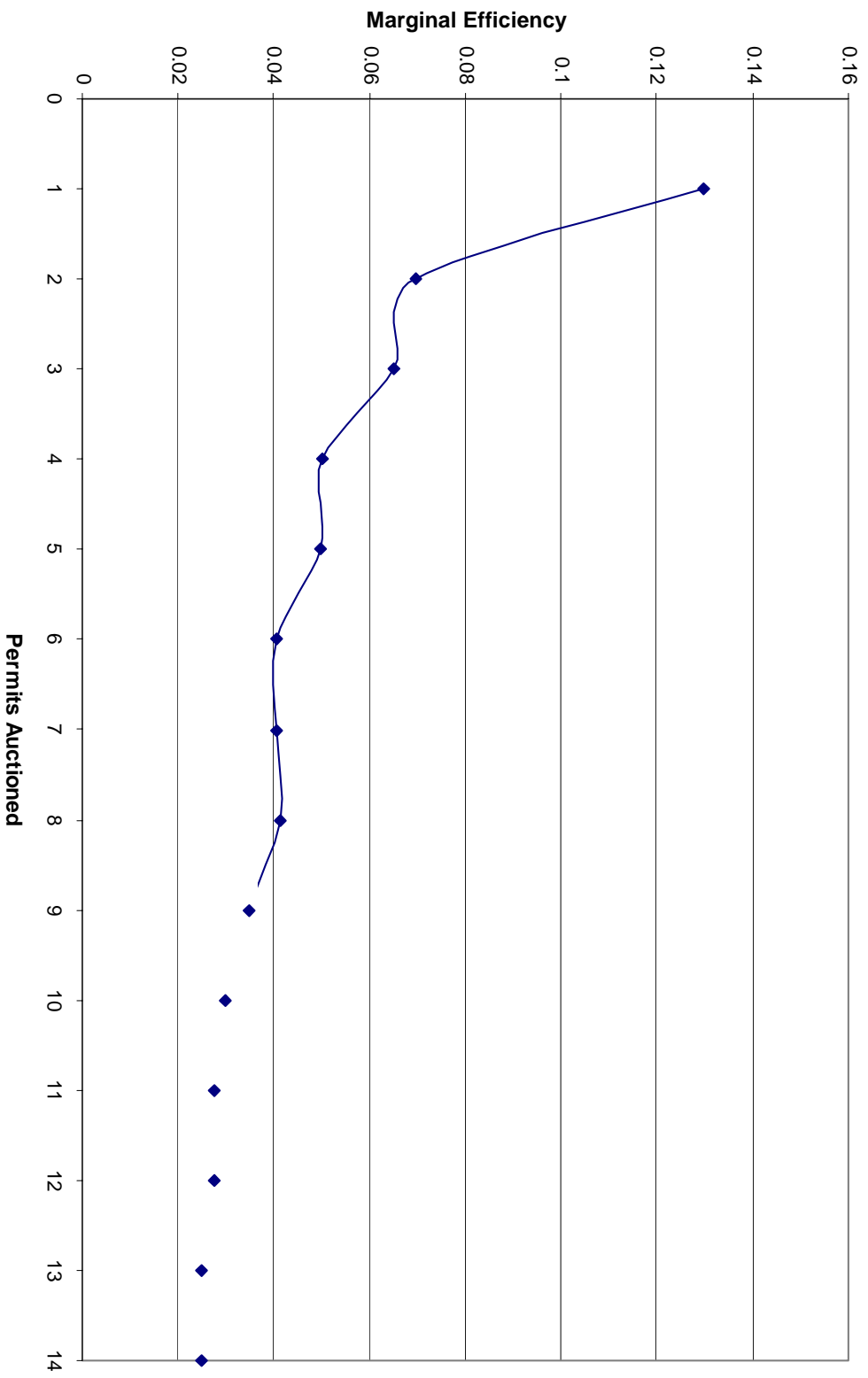
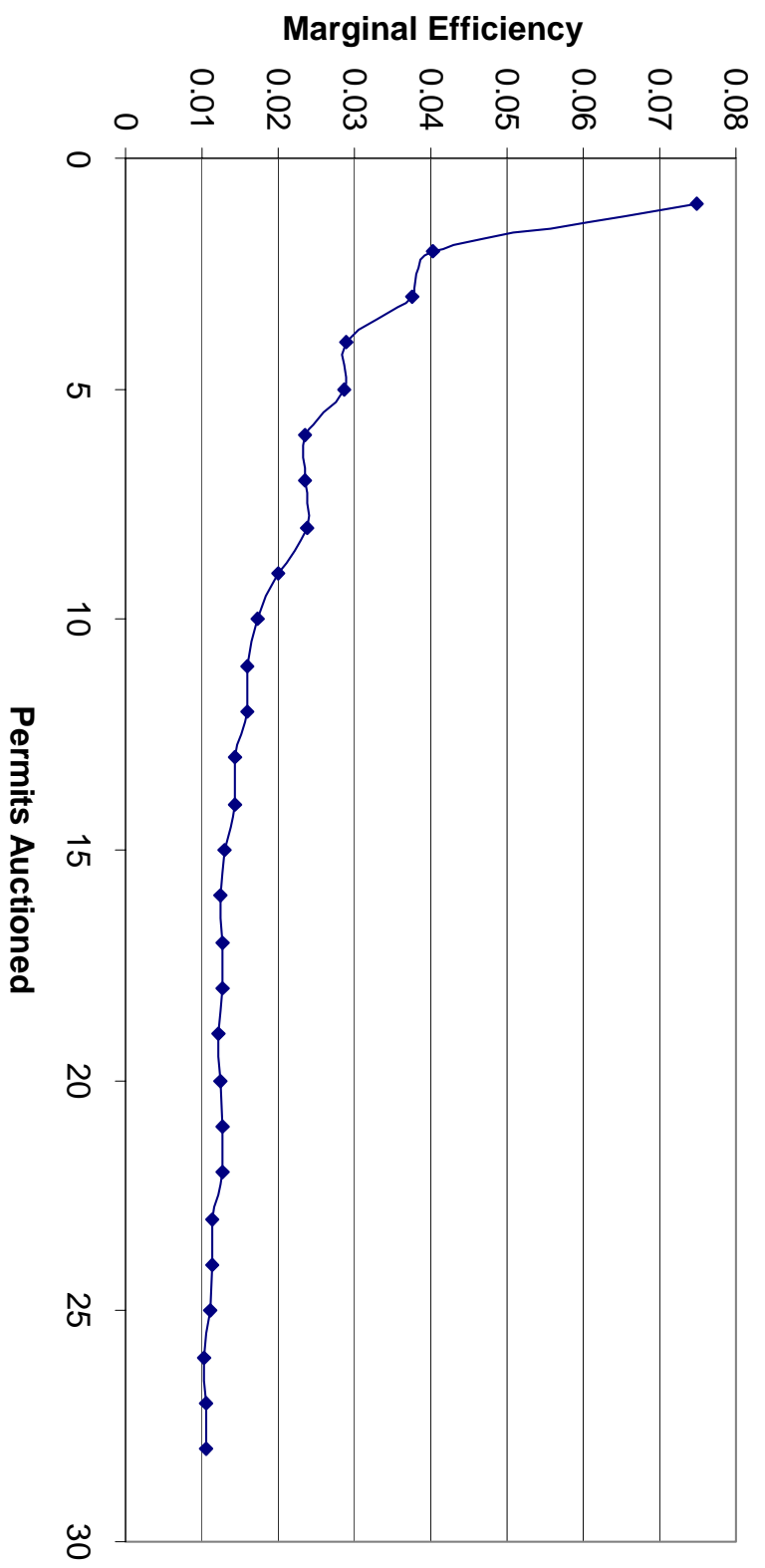


Figure 2 Marginal Efficiency of Auction/Lottery Combination



**Figure 3 Marginal Efficiency, 28 Feasible Permits**



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