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Does Evolution Solve the Hold-up Problem?

Tore Ellingsen
Department of Economics, Stockholm School of Economics
Stockholm, Sweden

Jack Robles
Department of Economics, University of Colorado at Boulder
Boulder, Colorado

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Center for Economic Analysis
Department of Economics

University of Colorado at Boulder
Boulder, Colorado 80309

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1 Introduction

2 Investment and Bargaining

$D_A \models \{V\ I() - x; V\ I()\}$

B

A

A D

B

A

$I; y\ x()$ A y $D \to D$

A $I \models I^*$ I

3 Evolution

A … B … *population* … N … $t \in \{\ldots\}$ … A … B … *beliefs* … A … B … B … A … B … x

Assumption 1 *(i) The pie division is small: V … $> -$. (ii) The population is large: V …*

adaptation

random mutation.

Θ

basin of attraction

μ *B* $\mu()$ μ'

μ' μ *single mutation neighborhood* μ $\mu' \in M$ $\mu()$ μ' μ

X *mutation connected set*

μ_1 $\mu_n \subset X$ $\mu_2, \ldots, \mu_{n-1}$

k ; ...; *n*; M $\mu() \cap BX\mu$ n

$$x^L = \max\{x \in D_B(I^*) \mid (V(I^*) - x)\frac{N-1}{N} - I^* \geq V(I) - I\}.$$

$$x^M = \max\{x \in D_B(I^*) \mid V(I^*) - x - I^* > V(I) - I\}.$$

$$x^M \leq x^L \leq x^M.$$

$$\begin{aligned} V^* - x^M (N-1)/N - I^* &= V(I^*) - x^M - I^* - (V(I^*) - x^M)/N \\ &\quad - (N-1)/N \\ &> V(I^*) - x^M - I^* - x^M/N \\ &\geq V(I^*) - x^M - I^* \\ &\geq V(I) - I; \end{aligned}$$

Proposition 3 *Let agents bargain according to the Nash demand game. The outcome ‰ is locally stable if and only if ‰ = {I*; V(I*) − x; x}, where x ≤ x^L.*

... $r(\mu;\mu')$ ... μ' ... $\mathcal{G}$ ... *resistance* ... *cost* ... $\mu \to \mu'$ ... $r(\mu;\mu')$ ... μ-*tree*, Γ ... $\mathcal{G}$ ... $\mu' \neq \mu$ ... μ ... Γ ... *stochastic potential* ... μ ... μ ...

Theorem 1 *An equilibrium* μ *is stochastically stable if and only if no other equilibrium has lower stochastic potential.*

... $I = I^*$ ... $y = V(I^*) - x$ ... B ... x ... $x < x^{NBS}$ ... $V(I^*)$ ... x ... $x > x^{NBS}$ ... x ... $r(x)$ ... B ... A ... $(I, V(I) - \ldots)$ ... $(I^*, V(I^*) - \ldots)$ ... $r(x)$ ... $(I^*, V(I^*) - x, x)$ ... $(I, V(I) - \ldots)$ ...

$r\ x() > r\ x()$ ... $x > x^L$ ...

... $x^M > x^{NBS}$ ...

$r\ x^L() \geq r\ x^L - -()$; ...

$$- \longrightarrow\ - \longrightarrow ::: \longrightarrow x^L - - \longrightarrow x^L;$$

... $r\ x^L() \geq r\ x^L - -()$ ...

$$x^L \longrightarrow - \longrightarrow\ - \longrightarrow ::: \longrightarrow x^L - -:$$

[illegible]

Appendix: Proofs

[illegible] Q [illegible]

Lemma 1 *Let* $z_1 < z_2 ::: < z$ *be demands in* $D\ I()$ *for some* $I \in \Psi$*. Assume that the set of demands following* I *for agents in the relevant population is* $\{z_l\}_l$

B [illegible] $I;y$ [illegible] A [illegible] $I;y$ [illegible] μ' [illegible] □

Lemma 4 *Let* μ' (‰ $\mu'(\,)$ |— $\{ I';y';x'(\,)\}$) *be an equilibrium. If* I ∠ I' *and* $y - I \geq y' - I'$, *then the population can get from* μ' *to an equilibrium* μ *with* ‰ $\mu(\,)$ |— $\{ I;y;x(\,)\}$ *through a sequence of single mutation transitions.*

[illegible] μ' [illegible] B [illegible] A [illegible] I [illegible] y [illegible] j [illegible] B x $I(\,)$ |— x |— V $I(\,) - y$ [illegible] A [illegible] $I;y(\,)$ [illegible] A [illegible] x $I(\,)$ |— x [illegible] j [illegible] B $y - I > y' - I'$ [illegible] $I;y(\,)$ [illegible] μ [illegible] ‰ $\mu(\,)$ |— $\{ I;x;y(\,)\}$ $y - I$ |— $y' - I'$ [illegible] μ_1 [illegible] ‰ $\mu_1(\,)$ |— $\{ I;y;x(\,); I';y';x'(\,)\}$ [illegible] μ [illegible] ‰ $\mu(\,)$ |— $\{ I;y;x(\,)\}$ □

[illegible] μ [illegible] ‰ $\mu(\,)$ [illegible] ‰ $\mu(\,)$ |— $\{ I;V\ I(\,) - x;x(\,)\}$ I ∠ I^* [illegible] $V\ I^*(\,) - - - I^* > V\ I(\,) - x - I;$ [illegible] μ^L [illegible] ‰ $\mu^L(\,)$ |— $\{ I^*;V^* - -;-(\,)\}$ I |— I^* $x > x^L$ [illegible] $V\ I(\,) - - - I \geq V\ I(\,) - x - I$ [illegible] μ [illegible] ‰ $\mu(\,)$ |— $\{ I;V\ I(\,) - -;-(\,)\}$ [illegible] μ^L [illegible] ‰ $\mu^L(\,)$ |— $\{ I^*;V^* - -;-(\,)\}$ [illegible] I |— I^* $x > x^L$ [illegible] $V\ I(\,) - - - I < V\ I(\,) - x - I$

$I; V \; I$

$\{ I^*; y; x \}$ $x \le x^L$

μ_1

$I \le V \; I$

$V \; I^* - x^L \; N - I = N - I^* \le V \; I^* - x \; N - I = N - I^*$

$x \; \mu_i$

A ... $I; V - -; -)$ ... B ...

$x^L \in \{x^M; x^M - -\}$:

Lemma 5 *The number of mutations required to get from an equilibrium with outcome $I^*; y; x)$ with $x \leq x^L$ to an equilibrium with outcome $I; V - -; -)$ is $r\ x) | - \ \{r | r > N\ - \frac{\hat{V} - \delta - \hat{I} + I^*}{V^* -})\}$.*

... r ... B ...

$$\frac{N - r}{N} V^* - x) - I^* < V - I - -;$$

... r ... □

Lemma 6

(i) If µ is an equilibrium with outcome $I^*; y; x)$ and $x <$ $\{x^M; x^{NBS}\}$;

the easiest transition away from ‰ µ) Sol...082TΩ0.9482 2 2 2F7283m)-343(with)-38326

$x > x^L$

$x \; x^L \; x^M \; x^M < x^{NBS}$;

$r\,x\,\geq r\,x$ $V^* - x^M - - \leq V - - - I$ □

$x \; x^M > x^{NBS}$

$I; V - -; -$ $I^*; y$ $-; x - -$

$I^*; y; x$ y $V^* - x$ $I^*; y'; x'$

Lemma 7 *From an outcome $I^*; y; x$ the easiest transition in which investment is at all times e–cient, but which ends with difierent demands, is to an outcome $I^*; y'; x'$ where $x \; x - -; x \; -; -;$ or $V^* - -$.*

□

Lemma 8

(ii) If x ⊢ *– then moving from x to*

[illegible] $x^L > x^{NBS}$ [illegible] Θ^1 [illegible] Γ^1 [illegible] $x < x^{NBS}$ [illegible] μ, μ' [illegible] $x^{NBS} < x \leq x^L$ [illegible] $r(x) < r(x - \delta)$ [illegible] $x < x^{NBS}$ [illegible] $r(x) > r(x - \delta)$ [illegible] $x > x^{NBS}$ □

[illegible]

Lemma 11 *Let surplus be divided by the ultimatum game. The component with the subgame perfect outcome, $(I^H, V^H - x^{\max}(I^H), x^{\max}(I^H))$, is a subset of the unique locally stable set.*

[illegible] $\exists \mu^H$ [illegible] $\mu^H \in T(\mu)$ [illegible] $\{(I^H, V^H - x^{\max}(I^H), x^{\max}(I^H))\}$ [illegible] $\{(I, V(I) - x, x)\}$ [illegible] $x^{\max}(I')$ [illegible] $I' \neq I$ [illegible] $x^{\max}(I)$ [illegible] $x^{\max}(I)$ [illegible] I^H [illegible]

□

Lemma 12 *Let surplus be divided by the ultimatum game. Agents in population A receive a payoff of at least $V^H - I^H - x^{\max}(I^H)$ in every equilibrium.*

[illegible] I^H [illegible] I^H □

Lemma 13 *Let surplus be divided by the 'ultimatum' game. If $V(I) - I - x \geq V^H - I^H - x^{\max}(I^H)$, then there exists an equilibrium μ such that $\mu \in \Theta^L$ and $\%(\mu) = (I, V(I) - x, x)$.*

6 References

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