

# DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 00-01

Does Evolution Solve the Hold-up Problem?

Tore Ellingsen

*Department of Economics, Stockholm School of Economics  
Stockholm, Sweden*

Jack Robles

*Department of Economics, University of Colorado at Boulder  
Boulder, Colorado*

January 2000

Center for Economic Analysis  
Department of Economics



University of Colorado at Boulder  
Boulder, Colorado 80309

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# **Does Evolution Solve the Hold{up Problem?**

# 1 Introduction

pp pp rrl pb \_

pg pl pl r<sup>k</sup>r

ll r f r p p l r k r g p p g g m r b l p  
 p b m l ll r l p k r p p b f b r g p p g  
 g m b p q a l r m b b r k r p p b r p  
 b p j a p b k r g p p g m b b p r l  
 r p g p f r r b b k b f ll p g a p  
 • \W b f b r g p p g g m b m l l g m r f a l r r'  
 • \W b f m l b r l p p b p g m r f p'

l p p p r p g m l p l b m l r b l p  
p p l f r g m r f l r p f r m r g p p g  
r m p b l p b r r f g m r f  
p m p b l p l r m l r g p p g g m  
b p g m r f a l r m p m l p l m m g m p  
b b b p r p m r r b r l m b r k p g  
r p r r l r p g m r f p b g m r b r k p g  
l m l f b l r l m b p r b l p r  
p m r b m g r b p b r n g r p r b l p p r l l  
b b l r l p p p l b p r b l m k r l l p  
p m p b l b m b l l m p b r  
m p l W n g b p p m p l l  
l b g b r m l f r g p p g g m b r l  
g g b g p r l r p l m g b r f b p b r p p  
p p p p g m r f p b W l p p l  
k m p m l l p a b m p b  
b r b p b p p p g m r f p r p m b  
l b l l n g r f b p g p r l f r b  
r g p p g g m r r p r m p l p r  
p l f b b l r l m n g b r p p r r g p p g g m  
p m p p m p  
b l r p l l p r b r l p g p r p l  
p W r p n g b p f r r p p b r p b b  
p r l r b r f b r l b p r p k  
b b r n g r p r l b l l p m r g  
f r l l b r r p l l r p b b p m p p r  
k p b r b p r r p p p f r l a l r m  
r b m m k b f r r p p r g m p b b r n g  
r p r b l r a r b r p b r p l b r b p  
b r n g r p r b l p r p b p a l r m b b m k  
b r l l f p m p r p k r b p r p m



## 2 Investment and Bargaining

$\mathcal{W} = \{ \omega \in \mathbb{R}^N \mid \omega_i \geq 0, \sum_{i=1}^N \omega_i = 1 \}$

$\Psi = \{ \psi = (\psi_0, \psi_1, \dots, \psi_N) \mid \psi_i \in [0, 1] \}$

$V(\omega) = \sum_{i=1}^N \omega_i v_i$

$\mathcal{W} \times \Psi$

llr p p pl b lpp DA  $\vdash \{V I\} - x; V I \} b r b$   
 $r$  lpp p p lpp pg B  $r$  p b p lpp  
q lpp r q pg r b llr f mpp f r A p  
p m r p r p p b gm l p m r p b  
f ll l r A b p mpp p D b p l p p  
p b r g p p g gm b Bb r m r p g p  
b l p m gm r l r A r r g f r b b l gm  
p r l y X() A m p f p p y, D  $\rightarrow$  D r b r b p r l  
p m r  
f r r p p g b l p r pl l p r b gm  
rf b q l r W b p b p m p p f ll b b  
p p gm q b r m l l f gm rf q l r p r  
l r b r r gm rf q l r p p g r p p m p  
p p pl A l b r g l  $\vdash$  / \* l



$\mathcal{S} \text{ gm } \mathcal{R}^f \text{ p m } r \text{ ll } p \text{ m } p \text{ b } b \text{ m } p \text{ gm}$   
 $p \text{ l } m \text{ l } p \text{ m } p \text{ b } l \text{ m } m \text{ gm } b \text{ p } l \text{ b}$   
 $p \text{ l } p \text{ r } \text{ ll } r \text{ p } b \text{ p } l \text{ b } r \text{ r } p \text{ f } l \text{ p } r$   
 $l \text{ p } \text{ ( } \text{ gm } \mathcal{R}^f \text{ p}$

### 3 Evolution

$\text{ } \text{ l } p \text{ r } p \text{ l } r \text{ f } p \text{ f } r \text{ m } p \text{ r } p \text{ g } b \text{ m } b \text{ l } p \text{ f}$   
 $\text{ } b \text{ b } r \text{ g } r \text{ r } p \text{ l } p \text{ r } p \text{ r } b \text{ m } r \text{ p } m$   
 $\text{ } p \text{ f } b \text{ b } r \text{ g } p \text{ r } m \text{ r } r$   
 $\text{ } b \text{ f } b \text{ l } p \text{ p } g \text{ m } p \text{ r } r \text{ p } p \text{ g}$   
 $\text{ } \text{ ) } p \text{ r } l \text{ b } p \text{ l } \text{ k } \text{ ) } p \text{ p } g \text{ l } \text{ ) } b \text{ p } p$   
 $\text{ } p \text{ f } r \text{ p } g \text{ m } l \text{ k } \text{ ) } p \text{ s } m \text{ l } p \text{ l } \text{ ) } p \text{ l}$   
 $\text{ } b \text{ r } f \text{ r } m \text{ r } k \text{ b } r \text{ } 7$

$r \text{ b } l \text{ r } r \text{ l } A \text{ p } B \text{ l } b \text{ r } \text{ population } \text{ f } N \text{ } b$   
 $r \text{ } t \in \{l; m\} \text{ r } l \text{ m } p \text{ p } \text{ f } g \text{ p } p \text{ l } p \text{ A } p$   
 $B \text{ p } p \text{ l } b \text{ p } m \text{ p } m \text{ r } g \text{ p } p \text{ g } g \text{ m } b \text{ f } r \text{ g } k$   
 $b \text{ m } g \text{ p } l \text{ b } l \text{ beliefs } b \text{ r } p \text{ p } \text{ f } p \text{ g}$   
 $p \text{ l } p \text{ r } r \text{ b } p \text{ r } g \text{ r } l \text{ r } b \text{ r } p \text{ ll } g \text{ p}$   
 $b \text{ r } l \text{ f } \text{ " } \text{ l } \text{ ) } p \text{ l } r \text{ A } l \text{ f } p \text{ r } p \text{ g } l \text{ r } B \text{ m } p$   
 $p \text{ l } \frac{3}{4} \text{ l } \text{ ) } p \text{ l } r \text{ B } l \text{ f } l \text{ r } A \text{ m } p \text{ b } \text{ "}$   
 $p \text{ } \frac{3}{4} \text{ r } r \text{ ) } l \text{ r } p \text{ p } b \text{ f } l \text{ m } p \text{ p } p \text{ b}$   
 $r \text{ p } p \text{ g } p \text{ b } p \text{ m } p \text{ l } \text{ f } r \text{ l } r \text{ p } g \text{ b}$   
 $l \text{ m } m \text{ g } m \text{ } \frac{3}{4} \text{ l } p \text{ p } l \text{ r } B \text{ m } p \text{ x}$   
 $\text{ } \text{ W } \text{ } \text{ } p \text{ l } \text{ [ } b \text{ p } l \text{ } \text{ } p$

**Assumption 1** (i) The pie division is small:  $V \text{ l } \text{ ) } > -$ . (ii) The population is large:  $V \text{ l } \text{ ) }^*$

♭           g p           ♭                           μ ♭ r           p  
r           l f           r           p f           r p p l p \ W ♭   ♭           p           z μ )  
          l (           p           r           g           l           p                           r p                           *adaptation*   ♭  
r p           p           r p p p           p                           *random mutation.*           p           r           p   ♭   f l  
l           p g           r           r           ♭   g p   ♭           p           ♭   p           f r           p ll           p g  
♭           l f           p           r           g           ♭           ll           p           p g r           p           p           p g           g p  
          r           z μ )                           ♭           l f                           p   ♭           r           p           l f           f ll           p g  
          p p           p           r           ♭           p                           μ r           p   ♭   p g           p           ♭                           r

$f: X \rightarrow X$  continuous map,  $\mu \in M(X)$ ,  $B(\mu)$  basin of attraction,  $\mu'$  single mutation, neighborhood of  $\mu$ ,  $\mu' \in M(\mu)$  mutation connected set,  $\mu_1, \dots, \mu_{n-1}$

$\mu \in M(\mu)$  neighborhood of  $\mu$ ,  $\mu' \in M(\mu)$  mutation connected set,  $\mu_1, \dots, \mu_{n-1}$

$\mu_1, \dots, \mu_{n-1}$

$\{x \in D_B \mid V(x) \geq V(I^*) - \frac{N-1}{N} (I^* - V(I^*))\}$

$$x^L \leftarrow \{x \in D_B \mid V(x) \geq V(I^*) - \frac{N-1}{N} (I^* - V(I^*))\}$$

$\{x \in D_B \mid V(x) \geq V(I^*) - \frac{N-1}{N} (I^* - V(I^*))\}$

$$x^M \leftarrow \{x \in D_B \mid V(x) \geq V(I^*) - \frac{N-1}{N} (I^* - V(I^*))\}$$

$x^M \leq x^L \leq x^M$

$$\begin{aligned}
 V^* - x^M &= N - I^* - V(I^*) - x^M - I^* - V(I^*) - x^M = N - N - I^* - I^* \\
 &> V(I^*) - x^M - I^* - x^M = N \\
 &\geq V(I^*) - x^M - I^* \\
 &\geq V(I^*) - I^*
 \end{aligned}$$

$\{x \in D_B \mid V(x) \geq V(I^*) - \frac{N-1}{N} (I^* - V(I^*))\}$

**Proposition 3** *Let agents bargain according to the Nash demand game. The outcome  $x_0$  is locally stable if and only if  $x_0 \in \{I^*; V(I^*) - x; x\}$ , where  $x \leq x^L$ .*

$\{x \in D_B \mid V(x) \geq V(I^*) - \frac{N-1}{N} (I^* - V(I^*))\}$

$\mu$  is stochastically stable if and only if no other equilibrium has lower stochastic potential.

**Theorem 1** *An equilibrium  $\mu$  is stochastically stable if and only if no other equilibrium has lower stochastic potential.*

$\mu$  is stochastically stable if and only if no other equilibrium has lower stochastic potential.

$$\begin{array}{c}
\begin{array}{c}
p \ b \ b \ r \ X \rangle > r \ X \rangle \ b \ p \ r \ X > X^L \\
p \ r \ p \ p \ p \ p \ r \ p \ r \ b \ X^M > X^{NBS} \ p \ \rangle \ \parallel \ r \\
b \ p \ l \ \{ \ b \ b \ p \ p \ g \ p \ p \ p \ g \ l \ \} \ b \ r \ \{ \\
r \ X^L \rangle \geq r \ X^L - \rangle \ b \ p \ p \ p \ p \ r \ p \ r \ g \ p \ \}
\end{array} \\
\longrightarrow \longrightarrow \dots \longrightarrow X^L \longrightarrow X^L; \\
b \ l \ \{ \ r \ X^L \rangle \geq r \ X^L - \rangle \ b \ p \ p \ p \ p \ r \ p \ r \ g \ p \\
X^L \longrightarrow \longrightarrow \dots \longrightarrow X^L \longrightarrow
\end{array}$$

p p p b p b r p p p p p l l r p<sup>8</sup>  
l s p b r g p p g p r p g b r l f b  
l p p g p p /<sup>H</sup> b b V /<sup>H</sup> - /<sup>H</sup>

$\mathbb{W} \setminus \{z_i\} \cap \mathbb{Q} \neq \emptyset$

## Appendix: Proofs

$\mathbb{W} \setminus \{z_i\} \cap \mathbb{Q} \neq \emptyset$

**Lemma 1** Let  $z_1 < z_2 < \dots < z_n$  be demands in  $D \setminus \{z_i\}$  for some  $i \in \Psi$ . Assume that the set of demands following  $i$  for agents in the relevant population is  $\{z_i\}_i$



$p$   $p$   $p$   $l$   $p$   $l$   $ll$   $g$   $p$   $p$   $b$   $r$   $l$   $p$   $b$   $p$   $k$   
 $l$   $b$   $p$   $p$   $V$   $l$   $l$   $-z$   $p$   $p$   $p$   $p$   $g$   $p$   $l$   $ll$   $p$   $p$   $k$   
 $p$   $p$   $z$   $p$   $b$   $p$   $p$   $b$   $r$   $p$   $p$   $r$   $r$   
 $b$   $b$   $p$   $r$   $b$   $p$   $p$   $b$   $Q$   $p$   $r$   $pg$   $sp$   $M$   $p$   $N$   
 $p$   $r$   $l$   $y$   $r$   $p$   $p$   $x$   $b$   $r$   $l$   $r$   $p$   
 $f$   $ll$   $pg$   $l$   $sp$   $b$   $p$   $p$   $r$   $r$   $b$   $r$   $r$   $g$   $pg$   $l$   $p$   
 $ll$   $b$   $r$   $f$   $r$   $b$   $l$   $p$   $p$   $g$   $l$   $r$   $p$   $Q$   $pg$   $p$   $\square$   
 $r$   $b$   $p$   $r$   $r$   $r$   $p$   
 $\backslash$   $W$   $r$   $b$   $p$   $r$   $r$   $r$   $p$   
 $r$   $f$   $f$   $r$   $p$   $p$   $b$   $Q$   $p$   $pg$   $p$   $sp$   $pg$   
 $p$   $b$   $pg$   $l$   $b$   $l$   $f$   $\%$

$$\begin{array}{l} Bb \quad p \quad p \quad p \quad b \quad pg \quad r \quad g \quad p \quad r^f r^p \quad pb \quad pr \quad l \\ b \quad r \quad r \quad r \quad gp \quad l \quad pg \quad l; y \rangle \rangle \quad p \quad p \quad gp \quad p \quad l \quad p A \\ b \quad p \quad p \quad p \quad b \quad pg \quad b \quad r \quad g \quad b \quad r \quad r \quad r \quad p \quad g \quad l \quad r \quad p \\ b \quad p \quad pr \quad gp \quad l \quad pg \quad l; y \rangle \quad r \quad pg \quad b \quad r \quad r^f \quad b \\ g \quad l \quad r \quad p \mu' \quad p \quad b \quad p \quad p \quad \square \end{array}$$

**Lemma 4** Let  $\mu'$  ( $\% \mu' \rangle_{l-} \{l'; y'; x'\}$ ) be an equilibrium. If  $l \angle l'$  and  $y-l \geq y'-l'$ , then the population can get from  $\mu'$  to an equilibrium  $\mu$  with  $\% \mu \rangle_{l-} \{l; y; x\}$  through a sequence of single mutation transitions.

$$\begin{array}{l} r \langle \langle p \quad \mu' l \quad gp \quad p \quad l \quad p B \quad r^f \quad l \quad b \quad r \quad p \quad b \\ p \quad gp \quad p \quad l \quad p A \quad b \quad p \quad l \quad \parallel \quad p \quad y \quad b \quad p \quad l \quad b \langle f r \\ \parallel j \quad p \quad l \quad p B \quad x \quad l \rangle_{l-} \quad x \quad V \quad l \rangle - y \quad pg \quad gp \quad p \quad l \quad p \\ A \quad p \quad l \quad l; y \rangle \quad p \quad b \quad pg \quad l \langle f g \quad p \quad b \quad p \quad r \quad l \quad \parallel \\ gp \quad p A \quad b \quad \parallel \quad r \quad b \quad x \quad l \rangle_{l-} \quad x \quad r \quad \parallel j \quad p \quad l \quad p B \\ \langle y-l > y'-l' \quad b \quad p \quad b \quad r \quad r \quad p \quad l; y \rangle \quad b \quad b \quad b \quad \parallel \quad b \\ l \quad pg \quad b \quad p \quad g \quad l \quad r \quad p \mu \quad b \quad \% \mu \rangle_{l-} \{l; x; y\} \langle y-l > y'-l' \quad b \quad p \\ \parallel \quad gp \quad p \quad l \quad p \quad r \quad l \quad pg \quad r \quad p \quad p \quad r \quad p \\ g \quad l \quad r \quad p \mu_1 \quad b \quad \% \mu_1 \rangle_{l-} \{l; y; x; l'; y'; x'\} \quad p \quad l \quad p \langle \quad p \quad p \\ g \quad \mu \quad b \quad \% \mu \rangle_{l-} \{l; y; x\} \quad \square \end{array}$$

$$\begin{array}{l} r \langle \langle f \quad r \quad p \quad W \quad p \quad p \quad r \quad p \quad b \quad b \quad l \quad p \quad pg \\ f \quad r \quad p \quad g \quad l \quad r \quad p \quad b \quad p \quad p \quad p \quad \langle pg \quad b \quad r \quad p \quad b \quad r \\ r \quad p \quad p \quad g \quad l \quad r \quad p \quad b \quad p \quad p \quad b \quad b \quad \langle \quad b \quad r \quad gb \\ p \quad g \quad p \quad \langle pg \quad p \quad p \quad r \quad p \quad p \quad p \quad \rangle \quad b \quad b \quad l \quad p \quad p \\ p \quad p \quad r \quad f \quad r \quad p \quad p \quad p \quad \langle pg \quad b \quad b \quad r \quad \rangle \quad r \quad p \quad b \quad l \\ p \quad p \quad l \quad p \quad p \quad p \quad p \quad S \quad p \quad p \quad p \quad p \quad r \quad p \quad l \quad \mu \\ b \quad b \quad \% \mu \rangle \quad pg \quad p \quad \% \mu \rangle_{l-} \{l; V \quad l \rangle - x; x\} \langle l \angle l^* \quad b \quad p \\ V \quad l^* \rangle - - - l^* > V \quad l \rangle - x - l; \quad p \quad p \quad b \quad l \quad p \quad pg \\ p \quad g \quad l \quad r \quad p \mu^L \quad b \quad \% \mu^L \rangle_{l-} \{l^*; V^* \quad - \quad - \} \langle l \angle l^* \quad x > x^L \quad p \\ V \quad l \rangle - - - l \geq V \quad l \rangle - x - l \quad b \quad p \quad p \quad b \quad l \quad p \quad pg \quad p \\ g \quad l \quad r \quad p \mu \quad b \quad \% \mu \rangle_{l-} \{l; V \quad l \rangle - - \quad - \} \quad p \quad b \quad r \quad l \quad p \quad f \quad p \quad p \\ b \quad pg \quad b \quad l \quad p \quad p \quad g \quad l \quad r \quad p \mu^L \quad b \quad \% \mu^L \rangle_{l-} \{l^*; V^* \quad - \quad - \} \\ p \quad \parallel \quad p \quad gb \quad b \quad l \angle l^* \quad x > x^L \quad p \quad V \quad l \rangle - - - l < V \quad l \rangle - x - l \end{array}$$

g f ll pg p p p p f l b b ll l b p l f ll pg  
r f pg) l; V l) - - ) p l p f p p p l b  
s ) p r p μ b % μ) - { l\*; y; x) } p x ≤ x<sup>L</sup>  
b b ) pg) p \* p p p l p b 1 p W μ<sub>1</sub> b  
% μ<sub>1</sub>) - % μ) r l i / l\* V l) - Ω - l ≤ V l) - r - l < Ω Ω f Ω  
V l\*) - x<sup>L</sup> N - l) = N - l\*) ≤ V l\*) - x) N - l) = N - l\*) p l g p  
p l p A x) μ<sub>j</sub>



$$r(x) \geq r(x')$$

$$V^* - X^M \leq V - \dots / \square$$

$$f(x) \leq x^M > x^{NBS}$$

$$V^* - X^M \leq V - \dots / \square$$

**Lemma 7** From an outcome  $(I^*; y; X)$  the easiest transition in which investment is at all times efficient, but which ends with different demands, is to an outcome  $(I^*; y'; X')$  where  $x_{t+1} \leq x_t$  or  $V^* \leq V$ .

$$r(x) \geq r(x')$$

$$V^* - X^M \leq V - \dots / \square$$

**Lemma 8**

(ii) If  $x_{\downarrow}$  - then moving from  $x$  to

$x^{NBS} < x \leq x^L$   $\mu; \mu'_{\neq \delta}$   $x < x^{NBS}$   $\mu; \mu'_{-\delta}$   $x > x^{NBS}$   $\square$

Detailed mathematical derivation involving game theory concepts like  $\mu$ ,  $\mu'$ ,  $\delta$ , and  $x^{NBS}$ .

**Lemma 11** *Let surplus be divided by the ultimatum game. The component with the subgame perfect outcome,  $(I^H; V^H - x^{\max}, I^H; x^{\max}, I^H)$  is a subset of the unique locally stable set.*

$\mu \in T \mu^H$   $\{I^H; V^H - x^{\max}, I^H; x^{\max}, I^H\}$   $\{I; V(I) - x; X\}$   $x^{\max}$   $I^H$   $B$   $A$   $I^H$   $\square$

**Lemma 12** *Let surplus be divided by the ultimatum game. Agents in population A receive a payoff of at least  $V^H - I^H - x^{\max}, I^H$  in every equilibrium.*

$I^H$   $\square$

**Lemma 13** *Let surplus be divided by the 'ultimatum' game. If  $V(I) - I - x \geq V^H - I^H - x^{\max}, I^H$  then there exists an equilibrium  $\mu$  such that  $\mu \in \Theta^L$  and  $\mu \in \{I; V(I) - x; X\}$*

r f m m f r m m l p □  
 b f r f m g b m A b g r b p  
 b g p b b l a l r m b p b r r g l r p b b b  
 m r r p b l m m r m p b r b p  
 g l r p r r p b l r b f b r l m m  
 r f f r p r m m l l p l k b b b  
 p g l ll l b b s m l p l b p m g l b  
 b ll l □



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