A draft of INRIA lectures, May 1991

Wavelets, Multiresolution Analysis and Fast Numerical Algorithms

e e de N e n c e e

$$p_j \quad \sum_{i \quad j} \frac{q_i q_j}{| \quad i}$$

[illegible]
[illegible]
[illegible]
[illegible]

II.1 Multiresolution analysis.

$$V_n \subset \cdots \subset V \subset V \subset V \quad V \subset L(R)$$

II.2 The Haar basis

$$d_k^{j+} = \frac{}{\sqrt{}} \left({}_k^{j} - {}_k^{j} \right)$$

$$ {}_k^{j+} = \frac{}{\sqrt{}} \left({}_k^{j} + {}_k^{j} \right)$$

[illegible] e [illegible] nd [illegible] d [illegible] e [illegible] ec [illegible] nd [illegible] de [illegible] ned [illegible] y [illegible] e [illegible] ee [illegible] nd [illegible] nc [illegible] n [illegible] ed [illegible] n [illegible] e [illegible] j;k [illegible] j;k′ [illegible] j;k [illegible] j;k′ [illegible] nd [illegible] j;k [illegible] j;k′ [illegible] e e [illegible] e c [illegible] c e [illegible] c [illegible] nc [illegible] n [illegible] e n e [illegible] nd [illegible] j;k [illegible] j= [illegible] j − [illegible] e e en [illegible] n [illegible] n [illegible] e [illegible] n [illegible] e d [illegible] e n [illegible] n [illegible] nd [illegible] d [illegible] e e [illegible] n [illegible] y ec [illegible] e c e [illegible] e [illegible]

[illegible] y c n [illegible] de n [illegible] n n e [illegible] e [illegible]

$$[illegible] \int [illegible] y \quad [illegible]$$

[illegible] nd e [illegible] nd n [illegible] e ne [illegible] n [illegible] d [illegible] en [illegible] n [illegible] e [illegible] nd [illegible] C [illegible] de [illegible] n Zy [illegible] nd [illegible] nd [illegible] e d [illegible] d [illegible] e en [illegible] e [illegible] e dec y [illegible] en [illegible] e [illegible] nc [illegible] n [illegible] e d [illegible] nce [illegible] e d [illegible] n [illegible] e [illegible] n [illegible] e e [illegible] e en [illegible] n [illegible] n [illegible] n [illegible] e [illegible] n [illegible] e ne [illegible] e e c [illegible] e [illegible] e [illegible] en [illegible] y n e [illegible] d [illegible] e ne [illegible] e [illegible] y [illegible] e d [illegible] n [illegible] e [illegible] e e ne [illegible] C [illegible] de [illegible] n Zy [illegible] nd [illegible] e [illegible] y [illegible] e e [illegible] e [illegible]

$$| [illegible] | \le \frac{[illegible]}{| [illegible] - y|}$$

$$| [illegible]^{M}_{x} [illegible] | + | [illegible]^{M}_{y} [illegible] | \le \frac{C_M}{| [illegible] - y|^{[illegible] + M}} \quad [illegible]$$

[illegible] e $M \ge$ [illegible] Le [illegible] M [illegible] n [illegible] nd c n [illegible] de

$$[illegible]^{j}_{kk'} [illegible] \int\int [illegible]_{j;k} [illegible]_{j;k'} [illegible] \, dy \quad [illegible]$$

[illegible] e e [illegible] e [illegible] e [illegible] e d [illegible] nce [illegible] e [illegible] een $| [illegible] - [illegible]'| \ge$ [illegible] nce

$$\int [illegible]_{j;k} [illegible] \quad [illegible]$$

e [illegible] e [illegible]

$| [illegible] ; [illegible] y \quad f \; r \quad x$ [illegible]

[illegible]

II.3 Orthonormal bases of compactly supported wavelets

[illegible]

ec nd he h n y { − k Z n e

$$\int_{\mathbf{k}} Z + \; - \; Z + \; |\, e^{i\mathbf{k}} \; d$$

nd he e e

$$\int_{\mathbf{k}} Z \sum_{\mathbf{l}\, Z} |\, \perp \; e^{i\mathbf{k}} \; d$$

nd

$$\sum_{\mathbf{l}\, Z} |\, \perp$$

n e n

$$\sum_{\mathbf{l}\, Z}$$

an d c ee e n e c ed e $\in \mathbf{Z}$ e
e e $\{$ $_{j;k}$ $_{j=}$ $_j$ $-$ $_k$ $\mathbf{Z}$ $\mathbf{W}_j$
e n n e ec e c e ze n e c yn
n c c e nd e n e c c y ed
e e n n en

Lemma II.1 **Any trigonometric polynomial solution of (2.26) is of the form**

$$h \quad \tfrac{1}{2} \quad \perp e^{i} \quad e^{i}$$

where $M \geq$ **is the number of vanishing moments, and where is a polynomial, such that**

$$| \quad e^{i} \quad P \quad n \; \tfrac{1}{2} \quad n^{M} \; \tfrac{1}{2} \quad \tfrac{1}{2} \, c$$

where

$$P\, y \; \sum_{k}^{M} \; M - \perp \; ! \; y^{k}$$

and is an odd polynomial, such that

$$\leq P\, y \; y^{M} \quad \tfrac{1}{2} - y \quad \text{for} \quad \leq y \leq$$

and

$$_{y} \; h \; P\, y \; y^{M} \quad \tfrac{1}{2} - y \quad M \;)$$

e and

he e $\mathbf{j}$ $\mathbf{k}$ nd $d^{n}_{\mathbf{k}}$ y e e ed e d c ence he e d $\mathbf{n}$ $\mathbf{j}$ C n
nd ed y e y d e e

$$\{\ \mathbf{k}\} \longrightarrow \{\ \mathbf{k}\} \longrightarrow \{\ \mathbf{k}\} \longrightarrow \{\ \mathbf{k}\} \cdots$$

$$\searrow \qquad \searrow \qquad \searrow$$

$$\{d_{\mathbf{k}}\} \qquad \{d$$

e hen de ne $f_{\mathbf{m}}$ $f_{\mathbf{m}} - \mathbf{m} \cdot f$ he e $\mathbf{m}$ c h en h $\langle f_{\mathbf{m}} \quad \mathbf{M} \rangle$

M c h e n he de ed h n.

II.5 A remark on computing in the wavelet bases

e n n en $\mathcal{M}^{m}$ e ned n e c. y h n h e de ed cc cy y ec e y r ene m ence ec $\{\mathcal{M}^{m}_{r}\}^{m}_{m}$ M r

$$\mathcal{M}^{m}_{r+} \quad \sum_{j}^{m} \quad j r \mathcal{M}^{m}_{r} \; j \mathcal{M}^{j}$$

n h

$$\mathcal{M}^{m} \quad m \; \tfrac{1}{2} \; \sum_{k}^{k} \; k \; m \quad M -$$

c h ec $\{\mathcal{M}^{m}_{r}\}^{m}_{m}$ M e e en M n n en h e d c n h r e n nd h e n c n e r e d y N ce e ne e c n ed h e nc n e

e non nd d nd nd d fo

III.1 The Non-Standard Form

Le e n e

$$\mathbf{L} \quad \mathbf{R} \rightarrow \mathbf{L} \quad \mathbf{R}$$

e e ne $\mathbf{L}$ e n n ec n e n e ce $\mathbf{V}_\mathbf{j}$; $\in \mathbf{Z}$

$$P_\mathbf{j} \quad \mathbf{L} \quad \mathbf{R} \rightarrow \mathbf{V}_\mathbf{j}$$

$$P_\mathbf{j} f \qquad \sum_\mathbf{k} \langle f \quad \mathbf{j;k} \rangle \quad \mathbf{j;k}$$

nd e nd n n e e c e e e n

$$\sum_{\mathbf{j} \quad \mathbf{Z}} \quad \mathbf{j} \quad \mathbf{j} \perp \quad \mathbf{j} \quad P_\mathbf{j} \perp P_\mathbf{j}$$

e e

$$\mathbf{j} \quad P_\mathbf{j} \quad - P_\mathbf{j}$$

e ec e n e ce $\mathbf{W}_\mathbf{j}$ e e e c e e n en n e d e e

$$\sum_\mathbf{j} \quad \mathbf{j} \quad \mathbf{j} \perp \quad \mathbf{j} \quad P_\mathbf{j} \perp P_\mathbf{j} \qquad \perp P_\mathbf{n} \quad P_\mathbf{n}$$

nd e e ; e ne e en

$$\sum_\mathbf{j} \quad \mathbf{j} \quad \mathbf{j} \perp \quad \mathbf{j} \quad P_\mathbf{j} \perp P_\mathbf{j} \qquad \perp P_\mathbf{n} \quad P_\mathbf{n}$$

e e $\sim$ P P d e z n e e n e ne e n n nd ec e e n c n n d e en e

e n n nd d e e en n ee e e c n e

$$\{A_\mathbf{j} \quad B_\mathbf{j} \quad \mathbf{j}\}_{\mathbf{j} \quad \mathbf{Z}}$$

c n n e ce $\mathbf{V}_\mathbf{j}$ nd $\mathbf{W}_\mathbf{j}$

$$A_\mathbf{j} \quad \mathbf{W}_\mathbf{j} \rightarrow \mathbf{W}_\mathbf{j}$$

$$B_\mathbf{j} \quad \mathbf{V}_\mathbf{j} \rightarrow \mathbf{W}_\mathbf{j}$$

$$j \quad \mathbf{W}_j \to \mathbf{V}_j$$

he e he e $\{A_j \; B_j \;\; j\}_{j \; \mathbf{Z}}$ e de ned A_j j j B_j j P_j nd j P_j j

he e $\{A_j \; B_j \;\; j\}_{j \; \mathbf{Z}}$ d ec e de n n he e n

$$j \quad \begin{matrix} A_{j+} & B_{j+} \\ j+ & j+ \end{matrix}$$

he e e j P_j P_j

$$j \quad \mathbf{V}_j \to \mathbf{V}_j$$

nd he e e e en ed y he $\times$ n n

$$\begin{matrix} A_{j+} & B_{j+} \\ j+ & j+ \end{matrix} \quad \mathbf{W}_{j+} \oplus \mathbf{V}_{j+} \to \mathbf{W}_{j+} \oplus \mathbf{V}_{j+}$$

he e c e n hen

$$\{\{A_j \; B_j \;\; j\}_{j \; \mathbf{Z} \; j \; n \;\; n}\}$$

he e n P_n P_n he n e e n e hen n n nd he e e n zed e he ee e nd

Le e he n e n

he e A_j de e he n e c n n he e e n y nce he ce $\mathbf{W}_j$ n n e e en he d ec n

he e B_j j n nd e e he n e c n e een he e nd c e nd eed he ce $\mathbf{V}_j$ c n n he ce $\mathbf{V}_{j'}$ h ee

he e j n e ed e n he e j

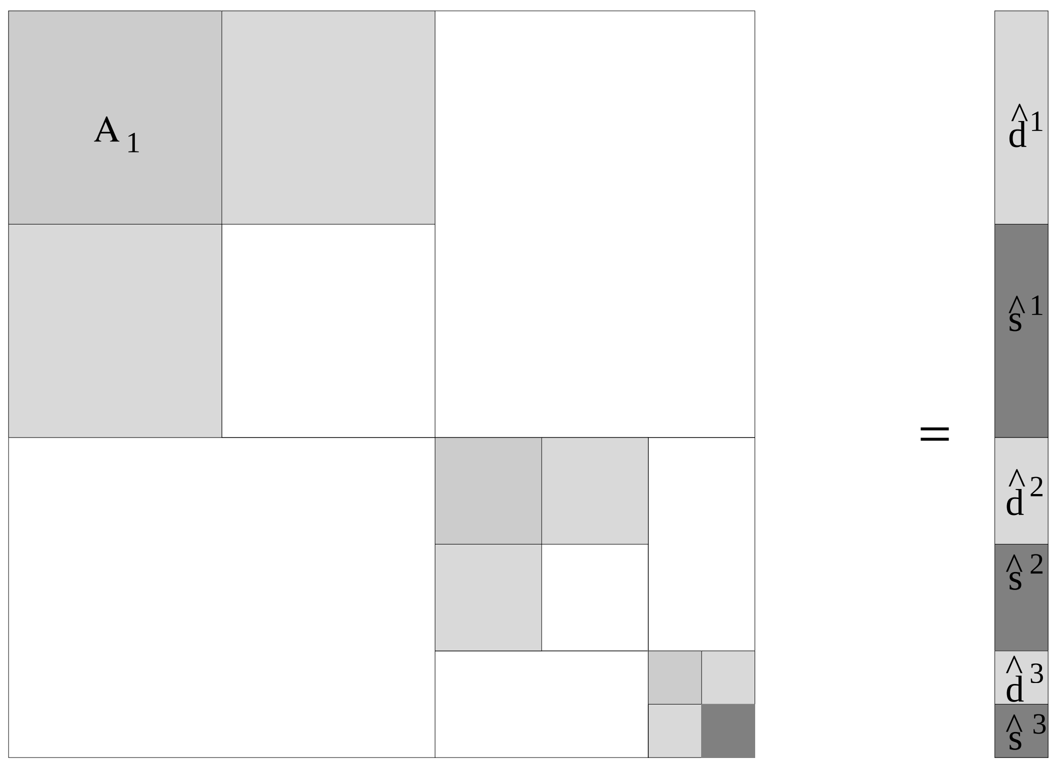
he e A_j B_j nd j e e e en ed y he ce j j nd j

$$j_{k;k'} \quad \int\!\!\int \quad y \;\; {}_{j;k} \;\; {}_{j;k'} \;\; l \; dy$$

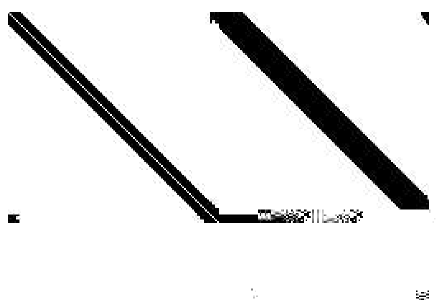
$$j_{k;k'} \quad \int\!\!\int \quad y \;\; {}_{j;k} \;\; {}_{j;k'} \;\; l \; dy$$

nd

$$j_{k;k'} \quad \int\!\!\int \quad y \;\; {}_{j;k} \;\; {}_{j;k'} \;\; l \; dy$$



A_1
=
$\hat{d}^1$
$\hat{s}^1$
$\hat{d}^2$
$\hat{s}^2$
$\hat{d}^3$
$\hat{s}^3$



e e j e e en ed y e j

$$\int\int_{k;k'}^{j} \quad j;k \quad j;k' \quad l \; dy$$

n en e c e c en k;k' $N -$ e e ed c n e

d ce

$$\sum_{i;l}^{j} \sum_{k;m}^{L} k \; m \; {}^{j}_{k+} \; i+ \; ;m+ \; l+$$

III.2 The Standard Form

e nd d n ned y e en n

$$V_j \quad \bigoplus_{j'>j} W_{j'}$$

nd c n de n e c e e $\{B_j^{j'} \quad {}_j^{j'}\}_{j'>j}$

$$B_j^{j'} \quad W_{j'} \to W_j$$

$$_j^{j'} \quad W_j \to W_{j'}$$

e e e c e e n en n e d e e

$$V_j \quad V_n \bigoplus_{j' \; j+}^{n} W_{j'}$$

n e e e $\{B_j^{j'} \quad {}_j^{j'}\}$ n e e n nd nd n dd n e c e e e e e $\{B_j^{n+}\}$ nd $\{{}_j^{n+}\}$

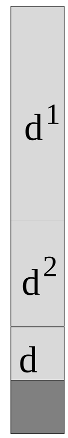
$$B_j^{n+} \quad V_n \to W_j$$

$$_j^{n+} \quad W_j \to V_n$$

n n n $_n^{n+}$ $_n$ nd B_n^{n+} B_n e n e e n e nd V n e d en n en e nd d e en n P P

$$\{A_j \; \{B_j^{j'}\}_{j'}^{n}{}_{j+} \; \{{}_j^{j'}\}_{j'}^{n}{}_{j+} \; B_j^{n+} \; {}_j^{n+} \quad {}_n\}_j$$ re e

e e e n zed e ee e nd e e e C de n Zy nd e d eratoj /R36 (3aV)Tj 7R360 Td (Figu65


d1
d2
d

=

e co p e on of ope o

[illegible]

the matrices ^{j}, ^{j}, ^{j} (3.16) - (3.18) of the non-standard form satisfy the estimate

$$|\ {}^j_{i;l}| + |\ {}^j_{i;l}| + |\ {}^j_{i;l}| \le \frac{C_M}{1 + |i - l|^{M+1}}$$

for all $|i - l| \ge 2M$.

[illegible] c [illegible] de [illegible] y n [illegible] e c [illegible] e [illegible] d [illegible] d e en [illegible] e [illegible] Le [illegible] e [illegible] d [illegible] d e en [illegible] e [illegible] ned y [illegible] e [illegible]

$$[\text{illegible}] \int e^{ix} [\text{illegible}] \int [\text{illegible}]\, dy$$

[illegible] e e [illegible] e d [illegible] e ne [illegible]

Proposition IV.2 **If the wavelet basis has M vanishing moments, then for any pseudo-differential operator with symbol of and of satisfying the standard conditions**

$$|\ {}_x \ [\text{illegible}] \le C \ ; \quad 1 + | [\text{illegible}]|^{+}$$

$$|\ {}_x \ [\text{illegible}] \le C \ ; \quad 1 + | [\text{illegible}]|^{+}$$

the matrices ^{j}, ^{j}, ^{j} (3.16) - (3.18) of the non-standard form satisfy the estimate

$$|\ {}^j_{i;l}| + |\ {}^j_{i;l}| + |\ {}^j_{i;l}| \le \frac{{}^j\, C_M [\text{illegible}]}{1 + |i - l| [\text{illegible}]^{+}}$$

for all integer i, l.

e [illegible] e [illegible] e e [illegible] ^{N} y [illegible] e e [illegible] $^{N;B}$ [illegible] ned [illegible] ^{N} y [illegible] e [illegible] ze [illegible] c e c en [illegible] ce [illegible] $^j_{i;l}$ $^j_{i;l}$ nd $^j_{i;l}$ [illegible] de [illegible] nd [illegible] d [illegible] $B \ge 2M$ [illegible] nd [illegible] e d [illegible] en [illegible] e [illegible] y [illegible] ee [illegible]

$$||\ ^{N;B} - \ ^N|| \le \frac{C}{B^M} \ [\text{illegible}]\ N$$

[illegible] e e C [illegible] c [illegible] n de e [illegible] ned y [illegible] e e ne [illegible] n [illegible] n e c [illegible] c [illegible] e [illegible] cc cy [illegible] c c [illegible] ed nd [illegible] e [illegible] e e [illegible] e [illegible] n [illegible] c [illegible] e [illegible] e nd d [illegible] B nd [illegible] de M [illegible] e [illegible] e c [illegible] en [illegible] c [illegible] nne [illegible] e de [illegible] ed [illegible] ec [illegible] c c [illegible] c [illegible] e ed M [illegible] ed [illegible] en B [illegible] e [illegible] c [illegible]

$$||\ ^{N;B} - \ ^N|| \le \frac{C}{B^M} \ [\text{illegible}]\ N \le$$

e . . e . c . . y . n . y . e nc . nd . nd e e e e . e . L . nd L e dec de . C. de n Zy nd e . nded

Theorem IV.1 (G. David, J.L. Journe) **Suppose that the operator (3.1) satis es the conditions (4.5), (4.6), and (4.16). Then a necessary and su cient condition for to be bounded on L is that n (4.24) and n (4.25) belong to dyadic BMO, i.e. satisfy condition**

$$\frac{1}{|J|}\int_J | \ldots|\, d \leq C$$

where is a dyadic interval and

$$\frac{1}{|J|}\int_J$$

e e . n e ee e nd e . e . . e y e . d e e . e e n e . e nc nd e e . y c ed h e ce c c n e n n . nd . d . nd . nd . y e ed de . e . e e n e e .

[illegible]

V.1 The operator d=dx in wavelet bases

[illegible]

[illegible] $\{ \cdot_k \}_k^L$

$$ [illegible] \sum_{i}^{L} [illegible] n \quad L - [illegible] $$

[illegible]

$$ [illegible] \quad L - [illegible] $$

[illegible]

$$ [illegible] - \perp - \sum_{n}^{L} [illegible] n $$

$$ [illegible] \perp [illegible] - - - \sum_{k}^{L=} [illegible] - [illegible] \perp - \sum [illegible] $$

[illegible]

$$ \sum_{k}^{L=} [illegible] $$

and hence [illegible] and [illegible]

[illegible]

$$ \sum_{k}^{k=} [illegible] - [illegible] \leq M - [illegible] $$

Since

$$ - [illegible] \leq M - [illegible] $$

[illegible]

[illegible]

$$r_i \; [illegible] \sum_{k} \; [illegible]$$

C[illegible] [illegible] $\sum_{k}$ [illegible]

$$r_l \; [illegible] \; r_l \perp \; [illegible] \; \sum_{n} r_l \, n \perp r_{l+n} \; [illegible] \in \mathbb{Z}$$

[illegible]

$$\sum_{l} \; [illegible] \; m \; - [illegible] \; m \perp \sum_{l} \; [illegible] \; M_l' \; [illegible]$$

where

$$M_l' \; [illegible] \int_{\mathbb{Z}} [illegible] \, d \; [illegible]$$

[illegible] Lenz [illegible]

$M \geq$ [illegible] then

$$| [illegible] | \leq C \; [illegible] \perp | [illegible]$$

[illegible] Le [illegible]

$$| [illegible] \leq C \; [illegible] \perp | [illegible]^{M + \log_2 B}$$

where

$$B \; [illegible] \; | \; [illegible]$$

[illegible] B $M -$ [illegible]

[illegible]

where

$$r \; \sum_{l} r_l e^{il}$$

$$r_{\text{even}} \; \sum_{l} r \; _l e^{il}$$

and

$$r_{\text{odd}} \; \sum_{l} r \; _{l+} e^{i \; l+)} =$$

N c n

$$r_{\text{even}} \; r \; r \; \perp$$

and

$$r_{\text{odd}} \; r \; r \; \perp$$

and n e n n

$$r \; h \; r \; r \; \perp \; - \; r \; r \; \perp$$

n y n e e

$$r \; r \; \perp \; r \; \perp$$

e n n e n r r nd n ene e n nd e n ene e e e en n d d en e n r_l nd e c n de e n e j de ned y e e c e c en n e ce $\mathbf{V}_j$ nd y c en y nc n f nce r_l^j $^j r_l$ e e

$$j \; \sum_{k \in Z} \; ^j \sum_{l} r_l f_{j;k} \; l \; \; j;k \; !$$

where

$$f_{j;k} \; l \; \; j= \; Z + \; f \; j \; - \; \perp$$

e n

$$f_{j;k} \; l$$

d

Remark 2

Examples.

$$C_M$$

$$\frac{M - }{M - }$$

[illegible] M

1 M

$$- \qquad --$$

and

$$r \quad -- \quad r \quad -$$

[illegible]

2 M

$$- \qquad - \qquad -$$

and

$$r \quad - \quad r \quad - \quad r \quad - \quad r_4 \quad -$$

3 M

$$- \qquad - \qquad - \qquad -$$

and

$$r \quad - \quad r \quad - \quad r \quad -$$

$$r_4 \quad - \quad r \quad - \quad r \quad -$$

4 M

$$- \qquad - \qquad - \qquad - \qquad -$$

and

$$r \quad - \quad r \quad - \quad r \quad -$$

$$r_4 \quad - \quad r \quad -$$

and

r r r

r_4 r r

r r r

r

C e c en M and M c n e c ed e c e nd n e e

Iterative algorithm for computing the coe cients r_l.

A y n e n nd e y n e e e r nd r nd e e ec e r_l e y e y nd e ed d e e c ce n z n e L ec e e e M c ed d y e c e c en $\{r_l\}_l$ e n e r_l $-r_l$ nd r

V.2 The operators d^n/dx^n in the wavelet bases

e e d d e n n nd d e e d^n d^n c e e y de e ned y e e en n n e ce $\mathbf{V}$ e y e c e c en

$$r_l^{(n)} = \int_{-\infty}^{+\infty} \ldots \frac{d^n}{dx^n} \ldots, \quad l \in \mathbf{Z}$$

e n e y

$$r_l^{(n)} = \int_{-\infty}^{+\infty} \ldots e^{il} \, d$$

e n e n e ee e e

	l	Coe cients l
M = 5	1	-0.82590601185015
	2	0.22882018706694
	3	-5.3352571932672E-

	l	Coe cients l
M = 8	1	-0.88344604609097
	2	0.30325935147672

Proposition V.2 **1. If the integrals in (5.52) or (5.53) exist, then the coe cients $r_l^{n)}$, $l \in \mathbf{Z}$ satisfy the following system of linear algebraic equations**

$$r_l^{n)} \quad {}^{n} \left[r_l \perp - \sum_{k}^{L=} {}_{k} \quad r_{l \; k+}^{n)} \perp r_{l+ \; k}^{n)} \right]$$

and

$$\sum_{l} {}^{n} r_l^{n)} \quad - n$$

where ${}_k$ **are given in (5.19).**

2. Let $M \geq n \perp$, where M is the number of vanishing moments in (2.16). If the integrals in (5.52) or (5.53) exist, then the equations (5.54) and (5.55) have a unique solution with a nite number of non-zero coe cients $r_l^{n)}$, namely, $r_l^{n)}$ / for $-L \perp \leq l \leq L -$. Also, for even n

$$r_l^{n)} \quad r_l^{n)}$$

$$\sum_{l} {}^{n} r_l^{n)} \quad n \quad n \; -$$

and

$$\sum_{l} r_l^{n)}$$

and for odd n

$$r_l^{n)} \quad -r_l^{n)}$$

$$\sum_{l} {}^{n} \quad r_l^{n)} \quad n \quad \leq l \leq L$$

$A \quad e \; M$

[illegible] en [illegible] e [illegible] e [illegible] ee [illegible] L [illegible] e [illegible] e [illegible] n [illegible] en [illegible] M [illegible] d [illegible] n [illegible] e [illegible] e [illegible] de e [illegible] hen [illegible] ee [illegible] e e [illegible] en [illegible] n [illegible] e [illegible] d de [illegible] ee [illegible] y [illegible] en [illegible] e [illegible] n [illegible] en [illegible] M

[illegible] e e [illegible] c [illegible] e c e c en [illegible] $r_l^{n)}$ [illegible] y [illegible] e ed [illegible] n e [illegible] en [illegible] e [illegible] e Le [illegible] de e [illegible] e e [illegible] n c [illegible] e [illegible] nd [illegible] d^n d^n d ec [illegible] y [illegible] e e [illegible] e [illegible]

$$r_l^{n)} \; Z \sum_{k \in Z} |. \perp [illegible]^n \perp [illegible] e^{il} \, d [illegible]$$

[illegible] e e [illegible] e

$$r [illegible] \sum_{k \in Z} |. \perp [illegible]^n \perp [illegible]$$

[illegible] e e

$$r [illegible] \sum_{l} r_l^{n)} e^{il}$$

[illegible] n [illegible] e e [illegible] n

$$[illegible]$$

n [illegible] e [illegible] nd [illegible] de [illegible] nd [illegible] e e en [illegible] nd [illegible] nd ce [illegible] n [illegible] e [illegible] e y e [illegible] e [illegible]

$$r [illegible]^n [illegible] r [illegible] \perp [illegible] r \perp [illegible]$$

Le [illegible] c [illegible] de [illegible] e [illegible] e [illegible] M [illegible] n e [illegible] c nc [illegible] n [illegible] d f n [illegible] f [illegible] d

$$M \, f [illegible] f [illegible] \perp [illegible]$$

N	[illegible]	[illegible]p
64	0.14545E+04	0.10792E+02
128	0.58181E+04	0.11511E+02
256	0.23272E+05	0.12091E+02
512	0.93089E+05	

VI.1 The Hilbert Transform

		Coe cients		Coe cients
	l	r_l	l	r_l
M = 6	1	-0.588303698	9	-0.035367761
	2	-0.077576414	10	-0.031830988
	3	-0.128743695	11	-0.028937262
	4	-0.075063628	12	-0.026525823
	5	-0.064168018	13	-0.024485376
	6	-0.053041366	14	-0.022736420
	7	-0.045470650	15	-0.021220659
	8	-0.039788641	16	-0.019894368

Example.

VI.2 The fractional derivatives

Example.

	Coe cients		Coe cients
l	I	l	I

M = 6	-7	-2.82831017E-06	4	-2.77955293E-02
	-6	-1.68623867E-06	5	-2.61324170E-02
	-5	4.45847796E-04	6-5	

[illegible]

VII.1 Multiplication of matrices in the standard form

[illegible]

VII.2 Multiplication of matrices in the non-standard form

n. y e e e n e

where

$$\sum_{j}^{n} \Big(A_j A_j + B_j \quad B_j \quad j + A_j B_j \quad j \quad j + \quad j A_j$$

and

$$n \quad n + \sum_{j}^{n} P_j \quad j B_j P_j$$

e e n e e c n n n e ce

$$A_j A_j + B_j \quad j \quad \mathbf{W}_j \to \mathbf{W}_j$$

$$B_j \quad j + A_j B_j \quad \mathbf{V}_j \to \mathbf{W}_j$$

$$j \quad j + \quad j A_j \quad \mathbf{W}_j \to \mathbf{V}_j$$

and e e n e

$$j B_j \quad \mathbf{V}_j \to \mathbf{V}_j$$

e e; n d d d

tot49 -410.315ac5 0 Td (36su8712.7097 0552

[illegible]

[illegible]

VIII.1 An iterative algorithm for computing the generalized inverse

[illegible]

[illegible]

$$A_{ij} \; [illegible]$$

[illegible] N [illegible] X_k [illegible]

Size $N \times N$	SVD	FWT Generalized Inverse	L_2-Error
128 × 128	20.27 sec.	25.89 sec.	3 1 · 10^{-4}
256 × 256	144.43 sec.	77.98 sec.	3 42 · 10^{-4}
512 × 512	1,155 sec. (est.)	242.84 sec.	6 0 · 10^{-4}
1024 × 1024	9,244 sec. (est.)	657.09 sec.	7 7 · 10^{-4}
...	...	...	...
$2^{15} \times 2^{15}$	9.6 years (est.)	1 day (est.)	

[illegible]

VIII.2 An iterative algorithm for computing the projection operator on the null space.

[illegible]

$$X_{k+} \quad X_k - X_k \; [illegible]$$

[illegible]

$$X \quad A\, A \; [illegible]$$

[illegible] A [illegible]

VIII.3 An iterative algorithm for computing a square root of an operator.

VIII.4 Fast algorithms for computing the exponential, sine and cosine of a matrix

[illegible]

X Co p n $F(u)$ n e e e e

[illegible]

IX.1 The algorithm for evaluating u²

[illegible] Let [illegible] $\in$ **L** **R** [illegible] $\mathbf{V}_j$, $j \in \mathbf{Z}$ [illegible]

$$P_j \quad j \in \mathbf{V}_j$$

[illegible]

$$- \quad \sum_{j}^{n} \left[P_j \quad - P_j \right] \quad \sum_{j}^{n} P_j \quad \perp P_j \, P_j \quad - P_j$$

[illegible] P_j $P_j \perp$ j [illegible]

$$- \quad \sum_{j}^{n} P_j \quad \perp \quad j \quad j$$

[illegible]

$$\sum_{j}^{n} P_j \quad j \quad \sum_{j}^{n} \quad j \quad j \quad n$$

[illegible]

[illegible]

$$j \quad k \quad j = \quad j \quad k$$
$$j \quad k \quad j = \quad j \quad k$$
$$j \quad k \quad j \quad k \quad j = \quad j \quad k$$

A [illegible]

$$j \sum^{n} \sum d^j_k \; j \; k \; \sum n \; k \; n \; k$$
$$j \quad k \; Z \qquad k \; Z$$

[illegible]

$$j \sum^{n} \quad j = \sum d^j_k \; j \; k \; j \; k \; j \sum^{n} \quad j = \sum d^j_k \; j \; k \quad n = \sum n \; k \; n \; k$$
$$j \quad k \; Z \qquad j \quad k \; Z \qquad k \; Z$$

[illegible]

$$d^j_k \quad j = + \; d^j_k \; j \; k$$
$$j \; k \quad j = \; d^j_k$$
$$n \; k \quad n = \; n \; k$$

[illegible]

$$j \sum^{n} \sum d^j_k \; j \; k \; j \sum^{n} \sum j \; k \; j \; k \; \sum n \; k \; n \; k$$
$$j \quad k \; Z \qquad j \quad k \; Z \qquad k \; Z$$

[illegible] **if the coe cient d^j_k is zero then there is no need to keep the corresponding average** $j \; k$ [illegible] d^j_k [illegible] $k \; d^j_k$ [illegible]

[illegible] c [illegible] c en [illegible] c [illegible] need [illegible] e [illegible] ed [illegible] y e ed ced [illegible] e y [illegible] e [illegible]
e [illegible] e

$$M^{\mathbf{j};\mathbf{j}'}_{\mathbf{www}} \; [illegible] \; \mathbf{j}'= \; Z \; + \; \mathbf{j} \; \mathbf{j}' \; [illegible] \; \mathbf{j} \; \mathbf{j}' \; \mathbf{k} \; \mathbf{k}' \; [illegible] \; {}_{j-j'}\mathbf{k} \; \mathbf{l} \; [illegible]$$

[illegible]

$$M^{\mathbf{j};\mathbf{j}'}_{\mathbf{www}} \; [illegible] \; \mathbf{j}'= M^{\mathbf{j}\;\mathbf{j}'}_{\mathbf{www}} \; - \; ' \; \mathbf{j} \; \mathbf{j}' \; - [illegible]$$

[illegible] e e [illegible] e [illegible] n [illegible] c n ed c [illegible] n [illegible] e n [illegible] e [illegible] c [illegible] c en [illegible] c n [illegible] e
[illegible] ence [illegible] e c [illegible] e c [illegible] c en [illegible] n [illegible] ec y [illegible] e d [illegible] nce r [illegible] $-$ [illegible]
e [illegible] een [illegible] e [illegible] e [illegible]

[illegible]

Remark. [illegible]

IX.2 The algorithm for evaluating F(u)

[illegible]

[illegible]

[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
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c e n SIAM Journal of Scienti c and Statistical Computing
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[illegible] M[illegible] e e [illegible] e ency c[illegible] nne dec[illegible] n [illegible] e[illegible] nd [illegible] e e[illegible] de [illegible] ec n c[illegible] e [illegible] C [illegible] n [illegible] n[illegible] e [illegible] M[illegible] e [illegible] c[illegible] c ence[illegible] Ne [illegible] n e[illegible] y

[illegible] Meye [illegible] Le c[illegible] c [illegible] en[illegible] e e[illegible] nde e[illegible] e[illegible] e[illegible] e[illegible] [illegible] en[illegible] d[illegible] e C [illegible] MA[illegible] n e[illegible] e [illegible] ne

[illegible] Meye [illegible] nc e d nce [illegible] de [illegible] e enne[illegible] e[illegible] e e[illegible] d [illegible] e [illegible] e [illegible] nw6ti3T