Iterated Spherical Means in Linearized Inverse Problems

Gregory Beylkin*

Abstract. We consider a representation of the function

[illegible]

in terms of the iterated spherical mean of $\hat{F}(p)$. Here, n is the dimension of the space. We also review applications of such a representation to linearized inverse problems and present [illegible]

$2\pi^{n/2}$ … is a point in R^n, … is a

If the support of $\hat{F}(p)$ is not restricted to the ball B_{2k} in (3.2), then (3.4) defines the and

$$Q_{LP}(x) = \frac{k^3}{\pi^2} \int \int |\nu-\mu| |f(k,\nu,\mu)|^2 e^{i(k\nu-k\mu)\cdot x} d\omega_\nu d\omega_\mu . \qquad (4.6)$$

Thus we obtai

where

$$\Psi(p,k,\nu) = \int U(x,k) e^{-ip\cdot x} dx\,,$$

$\delta(p-k\nu)$ represents the incident plane wave, p is a vector in R^2, ν is a unit vector in R^2 and

$$O_{LP}(x) = \frac{ik}{4\pi^3} \int_{|\nu|=1} \int_{|\mu|=1} (1-(\mu\cdot\nu)^2)^{1/2} |\mu\cdot\nu| \, e^{-ik|\mu\cdot\nu|y} \Psi_{sc}^{b}(y,\mu,k,\nu) e^{i(k\nu-k\mu)\cdot x} d\omega_\nu d\omega_\mu \, . \quad (5.9)$$

The formula (5.9) is a backpropagation inversion formula which was first obtained by A. J. Devaney [1] and is presented here in a slightly different form.

The case of Rytov approximation is analogous to Born approximation and can be found