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Department of Applied Mathematics, University of Colorado, Boulder, CO 80309-0526, United States

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> 0 $> 0,$ $\mathbf{r}^-,$ $> 0,$
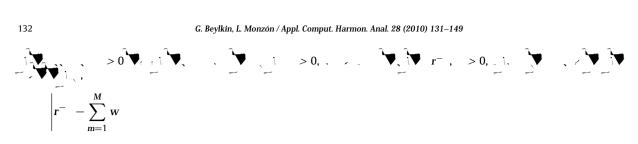
$$\left| \mathbf{r}^- - \sum_{m=1}^{M} w \right.$$

1. *Let us assume that* (4) *holds. For any* > 0 *and* $t_0 \in \mathbb{R}$, *we have*

$$\left| \int_{\mathbb{R}} f(t)\,dt - h \sum_{n \in \mathbb{Z}} f(t_0 + nh) \right| \leqslant \tag{6}$$

provided that the Fourier transform of f *satisfies*

$$\left| \hat{f}(\ ) \right| \leqslant c_1 e^{-q|\ |}, \tag{7}$$

for some positive constants c_1, q *and step size* $h \leqslant q/\quad (2c_1^{\ -1} + 1)$ *or, alternatively,*

$$\left| \hat{f}(\ ) \right| \leqslant \frac{c_2}{|\ |^q}, \quad \text{for } |\ | \geqslant R, \tag{8}$$

for some positive constants c_2, R, q *and step size* $h \leqslant \quad \{1/R, \ ^{1/q}(2c_2 \ (q))^{-1/q}\}$, *where* (q) *is the Riemann Zeta function.*

(5), $\qquad h \qquad \sum_{n \neq 0} |\hat{f}(\frac{n}{h})| \leqslant \qquad$ (7),

$$\sum_{n \neq 0} \left| \hat{f} \ \right\{$$

$$S_\infty(r) = \frac{h}{(\ )} \sum_{n \in \mathbb{Z}} e^{(t_0 + nh)} e^{-e^{t_0 + nh} r}. \tag{13}$$

$$h = h(\ ,\ )$$

$$\sum_{n \neq 0} \frac{|\ (\ + 2\ i\frac{n}{h})|}{(\ )} < \ . \tag{14}$$

3. *Given* > 0 *and* $0 < \ \leqslant 1$, *for any step size h such that*

$$h \leqslant \frac{2}{3 + \ (\ 1)^{-1} + \ ^{-1}}, \tag{15}$$

and any $t_0 \in \mathbb{R}$ *we have*

$$\frac{|r^- - S_\infty(r)|}{r^-} \leqslant \ , \quad \text{for all } r > 0, \tag{16}$$

where S_∞ *is given in* (13).

$S_\infty(r)$ (16)

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$\check{S}$

4. *For all r > 0,*

$$S_F(r) < S_\infty(r) < (\quad + 1)r$$

B ▼ M ▼ N, ▼ $f_{,r}(y)$ ▼ (9) (10) ▼ $r \in [\ , 1]$. ▼ $y_0 = (r^{-1})$. ▼ M ▼

$t_M \leqslant$ ▼ $f_{,r}(y)$ ▼ $(-\infty, t_M)$ ▼ $r \in (0, 1]$ ▼ $\mathcal{T}_M(r)$

$$\mathcal{T}_M(r) \leqslant \frac{r}{(\ )} \int_{-\infty}^{t_M} e^{-re^y + y} \, dy \leqslant \frac{1}{(\ )} \int_{-\infty}^{t_M} e^{-e^y + y} \, dy \tag{27}$$

$$= \frac{1}{(\ )} \int_0^{e^{t_M}} e^{-s} s^{-1} \, ds = 1 - \frac{(\ , e^{t_M})}{(\ )}, \tag{28}$$

$$(\ , x) = \int_x^{\infty} e^{-s} s^{-1} \, ds$$

▼ t_M ▼

N ▼

$$t_N \geqslant (\ ^{-1}\ ), \tag{29}$$

▼ fi

$$\mathcal{T}^N(r) \leqslant \frac{r}{(\ )} \int_{t_N}^{\infty} e^{-re^y + y} \, dy = \frac{1}{(\ )} \int_{re^{t_N}}^{\infty} e^{-s} s^{-1} \, ds,$$

$r \in [\ , 1]$ ▼

$$\mathcal{T}^N(r) \leqslant \frac{(\ , e^{t_N})}{(\ )}. \tag{30}$$

▼

$$\lim_{x \to 0} \frac{(\ , x)}{(\ )} = 1 \quad ▼ \quad \lim_{x \to \infty} \frac{(\ , x)}{(\ )} = 0,$$

▼ t_* ▼ t^* ▼

$$1 - \frac{(\ , e^{t_*})}{(\ )} = \ , \tag{31}$$

▼

$$\frac{(\ , e^{t})}{(\ )} = \ . \tag{32}$$

t_0, ▼ $(t_* = t_0)/h$ ▼ $(t^* - t_0)/h$ ▼ M_* ▼ N^* ▼ S_F, ▼ t_*, t^*, h ▼

7. For all $\ > 0$, $\ > 0$ and $1/e \geqslant \ > 0$, the solution t_* of (31) does not depend on $\ $ and satisfies

$$t_* \geqslant \frac{(1 + \ )}{} = \frac{1}{} \ , \quad + \ , \quad (1 + \ )^1. \tag{33}$$

The solution t^* of (32) has a weak dependence on $\ $ and satisfies

$$t^* \leqslant \ ^{-1} + \ , \quad ^{-1} + \ , \quad + \frac{1}{2}. \tag{34}$$

▼ t_* ▼ t^*, ▼ S_F ▼ 5. ▼ 7 ▼ A ▼ A.2.

▼ S_F ▼ (9) (10) ▼ $-\infty$ ▼ $+\infty$ ▼ e^{hn} ▼

(21) ▼ 3 ▼ n). A ▼ 16 ▼ (21) ▼

8. *For any* >0*, and* >0*, there exist a step size h and a positive integer M such that*

$$\left| e^{-xy} - G_e(x, y) \right| \leqslant \quad, \quad \textit{for } xy \geqslant \quad, \tag{41}$$

where

$$G_e(x, y) = \frac{hx}{2\sqrt{}} \sum_{j=0}^{M} e^{-x^2}$$

be an approximation of the kernel by Gaussians valid for $\leqslant$

A. $\quad$ $u(z, \quad)$ $\quad$ $z > 0,$ $\quad$ $u_{zz} + \quad u = 0$ $u(0, \quad) = u_0(\quad),$ $\quad$

$$u(z, \quad) = \int_{\mathbb{R}^d} \mathcal{P}(z, \quad - \quad) u_0(\quad) d \quad , \quad z \geqslant 0, \tag{56}$$

$$\mathcal{P}(z, \quad) = \frac{2}{\quad_d} \frac{z}{(z^2 + \| \quad \|^2)^{(d+1)/2}}$$

0 $\quad$ $3,$ $\quad > 0$ $z > 0,$ $\quad$ 0

$$S_\infty(z^2 + \| \quad \|^2) = \frac{zh}{\quad^{(d+1)/2}}$$

$0 \leqslant n \leqslant 2N$.

w_m (66) , $a > 0$ $\gamma_m = \frac{2N}{a} t_m$, $[0, 1]$ $h(x)$

$x \in [0, a]$,

$$\left| h(x) - \sum_{m=1}^{M} w_m e^{-\gamma_m x} \right| < \epsilon', \qquad (67)$$

h (66) (66) :

- B $(N+1) \times (N+1)$ $H_{kl} = h_{k+l}$ $h_n = h(a\frac{n}{2N})$, $0 \leqslant n \leqslant 2N$.
- $\mathbf{u} = (u_0, \ldots, u_N)$, $= \ldots$ (16, 22); fi $M+1$ $\sigma_0 \geqslant \sigma_1 \geqslant \cdots \geqslant \sigma_M$, $\sigma_M / \sigma_0 \approx \epsilon$, $M = \mathcal{O}(\epsilon^{-1})$ $M \ll N$.
- C γ_m $u($

$$x = \quad ^{-1}.$$

$$\tilde{t}^* = \quad ^{-1} + \quad ^{-1}. \tag{70}$$

$$> 1,$$

$$\frac{(\ ,x)}{(\ )} \leqslant \frac{e}{(\ )} x^{\ -1} e^{-x} \leqslant \quad ,$$

fi $\qquad$ (69)

$$x > d(\ -1) \tag{71}$$

$$d = e/(e-1) \approx 1.582, \qquad 14, \qquad x$$

$$x \geqslant d$$

[33] ... C ... 121 (7) (2004) 2866 2876.

[34] ... C ... 121 (14) (2004) 6680 6688.

[35] ... A ... C ... 20 (2) (1999) 699 718 (...).